

ONLINE SUPPLEMENT

WHEN PREDICTION MARKETS DISAGREE: CROSS-PLATFORM DISPERSION AND EQUITY VOLATILITY IN THE 2024 U.S. ELECTION

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This supplement contains (i) the full derivation of the conceptual framework summarized in Section 2 of the paper, (ii) the complete list of politically sensitive firms with their PAC-based classification, and (iii) supporting diagnostics referenced in the main text.

S.1 A parsimonious mechanism

The framework links cross-platform disagreement in election odds to the volatility of politically exposed equities. The goal is not a structural causal model; it clarifies why disagreement across venues can contain incremental information about volatility beyond the consensus probability, polling dispersion, or option-implied volatility, and it delivers the paper's empirical predictions.

S.1.1 Platform quotes and disagreement

Let π_t denote the latent market-relevant probability that the Republican candidate wins. Platform $j = 1, \dots, J$ posts

$$p_{j,t} = \pi_t + u_{j,t}, \quad (1)$$

where $u_{j,t}$ captures venue-specific information, clientele, liquidity, latency, and updating frictions. Let A_t denote the latent state of election-related ambiguity. Conditional on A_t ,

$$\mathbb{E}(u_{j,t} | A_t) = 0, \quad \text{Var}(u_{j,t} | A_t) = A_t, \quad \text{Cov}(u_{j,t}, u_{k,t} | A_t) = 0 \ (j \neq k). \quad (2)$$

This is deliberately reduced form: high A_t may reflect heterogeneous beliefs, slow information diffusion, fragmented liquidity, or differences in platform clienteles. Measuring disagreement as the cross-sectional dispersion of platform probabilities,

$$D_t^2 \equiv \frac{1}{J-1} \sum_{j=1}^J (p_{j,t} - \bar{p}_t)^2, \quad \bar{p}_t \equiv \frac{1}{J} \sum_{j=1}^J p_{j,t}, \quad (3)$$

equations (1)–(2) imply $\mathbb{E}(D_t^2 | A_t) = A_t$. Thus disagreement is an observable signal of the latent ambiguity state. The consensus probability $\bar{p}_t$ estimates the *level* of electoral beliefs; D_t estimates their *dispersion*. A single quote $p_{j,t}$ mixes π_t with $u_{j,t}$, whereas cross-platform dispersion uses multiple quotes on the same event to identify a second-moment object unavailable from any one platform's probability. Polling dispersion is lower-frequency and survey-based; option-implied volatility bundles election risk with macroeconomic and risk-premium components. Cross-platform prediction-market disagreement is a direct, high-frequency, market-based signal of disagreement about the election state.

S.1.2 Probability revisions and equity returns

Equity prices respond to revisions in the election probability. Let

$$\Delta\pi_{t+1} = \eta_{t+1}, \quad \mathbb{E}(\eta_{t+1} | \mathcal{F}_t, A_t) = 0, \quad \text{Var}(\eta_{t+1} | \mathcal{F}_t, A_t) = \sigma_{\pi,0}^2 + \kappa A_t, \quad (4)$$

with $\kappa > 0$: disagreement today forecasts a wider distribution of future probability revisions. For firm or portfolio i ,

$$r_{i,t+1} = \beta_i m_{t+1} + \lambda_i \Delta\pi_{t+1} + \varepsilon_{i,t+1}, \quad (5)$$

where m_{t+1} is a broad market shock, λ_i is political exposure ($\lambda_i > 0$ for firms that gain when the Republican win probability rises, $\lambda_i < 0$ otherwise, $\lambda_i \approx 0$ for non-political firms), and $\varepsilon_{i,t+1}$ is idiosyncratic. With the three components conditionally uncorrelated,

$$\text{Var}(r_{i,t+1} | \mathcal{F}_t, A_t) = \beta_i^2 \sigma_{m,t}^2 + \lambda_i^2 (\sigma_{\pi,0}^2 + \kappa A_t) + \sigma_{\varepsilon,i}^2, \quad (6)$$

so that, for politically exposed assets,

$$\frac{\partial \log \sqrt{\text{Var}(r_{i,t+1})}}{\partial A_t} = \frac{\lambda_i^2 \kappa}{2 [\beta_i^2 \sigma_{m,t}^2 + \lambda_i^2 (\sigma_{\pi,0}^2 + \kappa A_t) + \sigma_{\varepsilon,i}^2]} > 0 \quad \text{if } \lambda_i \neq 0. \quad (7)$$

S.1.3 Empirical implications

Prediction 1 (incremental to the consensus). Equity volatility depends on A_t , which D_t proxies for; disagreement therefore remains informative after controlling for the consensus probability and broad implied volatility. Table 1 of the paper *supports* this prediction: the disagreement coefficient remains economically large and statistically significant after controlling for realized market volatility, implied volatility, and the mechanical probability-uncertainty term.

Prediction 2 (selectivity). The partisan spread has exposure $\lambda_{\text{SPREAD}} = \lambda_R - \lambda_B$ (large), while an equal-weighted market portfolio has $\lambda_{\text{MKT}} = (n_R \lambda_R + n_B \lambda_B) / (n_R + n_B)$ (attenuated). By (7) disagreement predicts spread volatility more than market volatility, and more than a non-political benchmark with $\lambda \approx 0$. The selectivity tests *support* this prediction against both benchmarks: the interaction is significant against the broad market ($t = 3.9$; Table 1 of the paper) and against the clean non-political portfolio of utilities and consumer-staples firms ($t = 3.3$; Table 2 of the paper).

Prediction 3 (second moments, not direction). Since $\mathbb{E}(\Delta\pi_{t+1} | \mathcal{F}_t, A_t) = 0$, disagreement should predict absolute and squared returns more than signed returns. The data *predominantly support* this prediction: in Table 3 of the paper the absolute- and squared-return relations are an order of magnitude stronger in explanatory power than the signed-return relation, though a small, statistically significant positive signed component remains. The channel is predominantly, not exclusively, second-moment.

Prediction 4 (cross-sectional dispersion). With heterogeneous λ_i , $\text{Var}_i(r_{i,t+1} | A_t) \approx \text{Var}_i(\lambda_i)(\sigma_{\pi,0}^2 + \kappa A_t) + \text{Var}_i(\varepsilon_{i,t+1})$, so disagreement should widen the cross-sectional dispersion of firm returns. The evidence here is *mixed*: in Table S.3 the relation is positive and significant within Democrat-leaning firms but not robustly so within the Republican-leaning or pooled sets, and we report it as suggestive. The stronger effect within Democrat-leaning firms is consistent with the unusual mid-sample replacement of President Biden by Vice President Harris, which forced a rapid repricing across the Democratic coalition while the Republican nominee was stable, though we do not push this reading.

Prediction 5 (persistence). If $A_{t+1} = \rho A_t + v_{t+1}$, the effect on $\mathbb{E}[\text{Var}(r_{i,t+h} | A_t)]$ decays at rate ρ^h . The dynamics *support* this prediction: the local projections in Figure 2 of the paper and the VAR impulse response

in Figure 3 show a positive response that persists for roughly one trading day, consistent with a persistent ambiguity state.

Prediction 6 (platform heterogeneity). Writing $p_{j,t} = \pi_t + \tau_j z_t + e_{j,t}$ with $z_t \sim (0, A_t)$, venues with more heterogeneous τ_j (slower updating, different clienteles) load more heavily on A_t . This rationalizes why lower-price-discovery venues may carry more second-moment information, while keeping the interpretation cautious—liquidity, staleness, and clientele are alternative microfoundations. We read the evidence as *suggestive*: the binary decomposition in Table 4 of the paper, the platform diagnostics in Table 5, and the continuous Gonzalo–Granger-weighted measure in Table S.4 are consistent with this mechanism but do not discriminate sharply among the alternative microfoundations.

S.1.4 Competing interpretations

A purely market-wide news interpretation predicts similar effects on spread and market volatility; the selectivity result instead requires the effect to concentrate where $|\lambda|$ is large. A directional election-news interpretation predicts signed-return predictability; the second-moment channel predicts volatility, absolute, and squared returns. A purely mechanical stale-quote or liquidity-fragmentation interpretation can generate dispersion but does not by itself explain selectivity or persistence. The platform table in the paper reports platform staleness and update frequency to gauge how much of A_t reflects microstructure rather than ambiguity.

S.2 Firm classification

Table S.1 lists every politically sensitive firm with its sector, cumulative federal PAC contributions to Democratic and Republican recipients (\$ millions, OpenSecrets), the resulting Republican share, and the binary lean used in the paper. Republican-leaning firms have a Republican share above one-half on average; the paper’s tercile robustness check (Table 8) drops the middle third and forms the spread from the extreme thirds.

Table S.1: Politically sensitive firms and PAC-based classification

Ticker	Sector	Dem (\$m)	Rep (\$m)	Rep. share	Lean
UNH	health	1.4	1.3	0.48	BLUE
HON	misc	1.1	0.9	0.46	BLUE
RTX	defense	1.5	1.3	0.46	BLUE
LMT	defense	2.5	2.0	0.45	BLUE
LDOS	defense	0.8	0.6	0.44	BLUE
BA	transport	2.6	2.0	0.43	BLUE
NEE	energy	1.3	1.0	0.43	BLUE
F	transport	0.6	0.5	0.43	BLUE
NSC	transport	0.6	0.4	0.42	BLUE
BAH	defense	0.7	0.5	0.42	BLUE
NOC	defense	2.0	1.4	0.42	BLUE
T	tech	2.9	1.9	0.40	BLUE
GM	transport	1.4	0.9	0.39	BLUE
CMCSA	tech	2.7	1.7	0.38	BLUE

continued

Ticker	Sector	Dem (\$m)	Rep (\$m)	Rep. share	Lean
WPP	law	1.0	0.6	0.38	BLUE
PFE	health	1.4	0.8	0.37	BLUE
UAL	transport	1.7	0.9	0.34	BLUE
CNC	health	1.1	0.6	0.34	BLUE
AZN	health	0.9	0.3	0.27	BLUE
ORCL	tech	3.3	1.2	0.27	BLUE
MRK	health	1.4	0.4	0.24	BLUE
IBM	tech	2.8	0.9	0.24	BLUE
ACM	construction	0.9	0.2	0.20	BLUE
META	tech	4.5	0.9	0.17	BLUE
AMZN	tech	5.8	1.1	0.16	BLUE
MSFT	tech	11.3	1.8	0.14	BLUE
GOOG	tech	13.3	1.3	0.09	BLUE
DIS	tech	3.4	0.3	0.09	BLUE
AAPL	tech	4.8	0.3	0.05	BLUE
MASI	health	1.5	0.0	0.01	BLUE
WBD	tech	3.6	0.0	0.01	BLUE
ANET	tech	3.9	0.0	0.00	BLUE
NFLX	tech	10.0	0.0	0.00	BLUE
TWLO	tech	5.8	0.0	0.00	BLUE
ASAN	tech	50.8	0.0	0.00	BLUE
ET	energy	0.0	16.0	1.00	RED
CLR	energy	0.0	6.0	1.00	RED
SCHW	finance	0.1	18.5	1.00	RED
BTI	agribusiness	0.1	19.5	1.00	RED
OXY	energy	0.0	6.6	0.99	RED
DVN	energy	0.1	4.4	0.99	RED
MDC	construction	0.1	2.8	0.98	RED
RYAN	finance	0.4	19.9	0.98	RED
CVX	energy	0.6	8.9	0.93	RED
COP	energy	0.4	5.5	0.93	RED
MO	agribusiness	0.5	5.0	0.92	RED
BX	finance	5.1	40.9	0.89	RED
ABT	health	0.6	4.6	0.88	RED
TDG	transport	0.3	0.9	0.78	RED
TSN	agribusiness	0.2	0.7	0.76	RED
CACI	defense	0.2	0.4	0.72	RED
UNP	transport	0.5	1.1	0.71	RED
FDX	transport	0.9	2.0	0.69	RED
DE	agribusiness	0.2	0.5	0.67	RED
HD	misc	1.4	2.8	0.66	RED
PLTR	tech	1.3	2.6	0.66	RED
AMGN	health	0.4	0.7	0.63	RED
LUV	transport	0.6	0.8	0.58	RED

continued

Ticker	Sector	Dem (\$m)	Rep (\$m)	Rep. share	Lean
IP	agribusiness	0.3	0.4	0.58	RED
PSN	defense	0.2	0.3	0.57	RED
WMT	misc	0.9	1.1	0.55	RED
DAL	transport	1.4	1.7	0.54	RED
LHX	defense	1.1	1.3	0.54	RED
UPS	transport	1.5	1.7	0.54	RED
HII	defense	0.4	0.5	0.53	RED
BAESY	defense	0.5	0.6	0.53	RED
TM	transport	0.7	0.7	0.51	RED
AAL	transport	1.2	1.2	0.51	RED
GD	defense	1.1	1.1	0.50	RED
COIN	finance	4.3	3.0	0.42	RED
TSLA	tech	n/a	n/a	n/a	RED

S.2.0.0.1 Classification rule. The binary lean follows each firm's overall partisan giving profile from OpenSecrets—contributions to candidates *and* to ideological groups—so the displayed Republican share (which uses these full totals) reproduces the lean for all but two firms. Coinbase (COIN) gives heavily to both sides with no net tilt and is assigned to the Republican group; Tesla (TSLA) has no recorded corporate-PAC contributions and is assigned to the Republican group on qualitative grounds. Dropping both judgment cases leaves the baseline relation essentially unchanged ($\hat{\beta} = 15.8$, $t = 7.8$).

S.2.0.0.2 Non-political benchmark. The non-political portfolio is an equal-weighted basket of sixteen large, liquid firms from two sectors with negligible partisan tilt—regulated utilities and consumer staples—none among the 71 politically exposed firms. The constituents are, in utilities, Duke Energy (DUK), Southern (SO), Dominion (D), American Electric Power (AEP), Exelon (EXC), Xcel (XEL), Consolidated Edison (ED), and WEC Energy (WEC); and, in consumer staples, Procter & Gamble (PG), Coca-Cola (KO), PepsiCo (PEP), Colgate-Palmolive (CL), Kimberly-Clark (KMB), General Mills (GIS), Hershey (HSY), and Clorox (CLX). The selection rule is: large-capitalization names from these two low-salience sectors, disjoint from the political sample, priced at the same 5-minute frequency from DataBento; the S&P 500 ETF (SPY) serves as an additional broad-market benchmark. No sector is wholly free of macro-policy risk, but these firms carry near-zero *partisan* tilt, which is what the selectivity test requires.

S.3 Additional diagnostics

Table S.2 reports variance-inflation factors for the saturated specification and the Frisch–Waugh partial coefficient. Table S.3 reports the cross-sectional dispersion tests. Table S.4 reports the continuous Gonzalo–Granger-weighted disagreement measures. For completeness, the nine-market Gonzalo–Granger weights (adding Kalshi and Pinnacle over their shorter common window) are, in order, PolyMarket 0.30, Betfair 0.21, Pinnacle 0.22, Bovada 0.13, BetOnline -0.06 , William Hill 0.03, Unibet -0.02 , Everygame -0.06 , and Kalshi 0.24; PolyMarket and Betfair remain among the leading venues.

Table S.2: Variance inflation factors in the saturated specification

Regressor	VIF
disagree_sd	24.29
disagree_lag1	24.34
ln_mkt_vol	1.13
ln_vixy	1.54
prob_uncertainty	1.33
ln_spread_vol_lag1	1.18

Notes: VIFs from the saturated (“kitchen-sink”) regression. Contemporaneous and lagged disagreement are nearly collinear (pairwise correlation 0.979), inflating their variances; all other regressors have $VIF < 2$. Adding disagreement to the control-only model raises R^2 from 0.425 to 0.454 (incremental $R^2 = 0.029$); the Frisch–Waugh partial coefficient is 12.47 ($t = 6.78$), confirming multicollinearity rather than fragility.

Table S.3: Disagreement and cross-sectional return dispersion (hourly horizon)

Universe	Disagreement	t	N
Within RED	-1.450*	(-1.72)	1401
Within BLUE	2.350**	(2.42)	1401
All 71	0.577	(0.70)	1401
Non-political	0.717	(1.62)	1355

Notes: Hourly cross-sectional standard deviation of firm-level hourly net returns regressed on disagreement. Evidence is mixed: disagreement predicts greater dispersion within Democrat-leaning firms but not robustly for Republican-leaning or pooled sets. We report it as suggestive. HAC t -statistics. *** $p < 0.01$, ** $p < 0.05$, * $p < 0.10$.

Table S.4: Continuous Gonzalo–Granger-weighted disagreement

	(1)	(2)	(3) Joint
Leader-weighted	15.462*** (6.65)		10.778*** (3.55)
Lagger-weighted		18.104*** (5.94)	9.935** (2.53)
R^2	0.045	0.039	0.053

Notes: Disagreement re-weighted continuously by each platform’s price-discovery weight: leader-weighting loads on high- ψ venues, lagger-weighting on low- ψ venues. Both predict partisan-spread volatility; the lagger-weighted measure carries a somewhat larger univariate loading, consistent with the binary decomposition. HAC t -statistics.