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Optimum design of composite laminates for maximum buckling load capacity using simulated annealing

Ozgur Erdal, Fazil O. Sonmez *

Department of Mechanical Engineering, Bogazici University, Istanbul, Bebek 34342, Turkey

Abstract

This paper presents a method to find globally optimum designs for two-dimensional composite structures subject to given in-plane static loads for which the critical failure mode is buckling. The aim is to maximize the buckling load capacity of laminated composites. For this purpose an improved version of simulated annealing algorithm, which is direct simulated annealing (DSA), was utilized. Fiber orientation in each layer was taken as a design variable. A computer code was developed, and results were obtained for several load cases.

Keywords: Buckling strength optimization; Direct simulated annealing; Composite laminates; Optimal design

1. Introduction

Composite materials are mostly used in applications where their superior stiffness-to-weight or strength-to-weight ratios are critical. As a further advantage, configuration of a laminate, i.e. fiber orientation, ply thickness, stacking sequence, reinforcement geometry (continuous fibers, particulates, etc.), volume fraction of reinforcement, can be tailored to reduce its weight without compromising its performance, or improve the performance without increasing its weight. This can be achieved through a process of design optimization.

Optimization provides the engineers with a tool that is essential in finding the best design among countless number of designs. Design optimization of composite structures was observed to be a global optimization problem, with multiple local optima and complex design space. A deterministic algorithm, in which a monotonically decreasing value of an objective function is iteratively created, may stuck into any local optimum point rather than globally optimum one depending on the starting point. Therefore, its success depends on the choice of initial design. The usual approach is to employ the algorithm many times starting from different configurations with the hope that one of the initial positions be sufficiently close to the globally optimum configuration, and then to choose the lowest value as the globally optimum solution. Another disadvantage is that if the starting point is outside the feasible region, the algorithm may converge to a local optimum within the infeasible domain.

In order to find the absolute optimum of an objective function without being sensitive to the starting position, a global optimization method has to be employed in structural optimization problems. Stochastic optimization techniques are quite suitable in this respect. Among their advantages, they are not sensitive to starting point, they can search a large solution space, and they can escape local optimum points because they allow occasional uphill moves. The genetic algorithm (GA) and the simulated annealing algorithm (SA) are two of the most popular stochastic optimization techniques.

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* Corresponding author. Tel.: +90 212 359 7196; fax: +90 212 287 2456.

E-mail address: sonmezfa@boun.edu.tr (F.O. Sonmez).

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Researchers applied various optimization methods to a number of different design problems involving composite materials using design variables such as fiber direction, ply thickness, or stacking sequence. As one of the methods, enumeration, evaluating all possible designs, has a restricted applicability; for most problems it is computationally too expensive, for many impracticable. Genetic algorithms (GA) are well suited for stacking sequence optimization, and because of their random nature, they can produce alternative optima in repeated runs. The first application of the algorithm to a structural design problem was presented by Goldberg and Samtani [1], who solved a well-known weight minimization problem of a 10-bar truss structure. The GA algorithm was also applied to optimization of composite laminates. Sivakumar et al. [2] attempted to design a laminated composite plate in the presence of elliptical cutouts for optimum free-vibration response. Park et al. [3] used a GA for the optimal design of composite laminates which comprises four plies subject to various in-plane loading conditions. Fiber orientations of layers were taken as design variables and the objective function was formulated using Tsai-Hill failure criterion. Kristinsdottir et al. [4] presented a formulation to optimize the design of large composite panels when loads vary over the panel. The aim was to minimize the weight of the panel using the thickness and orientations of layers as design variables. They used a global optimization algorithm, improving Hit-and-Run (IHR). Antonio [5] applied the Newton-Raphson iterative procedure and the arc-length method to optimize geometrically nonlinear composite structures based on load–displacement control. Using the fiber orientations as design variables Savic et al. [6] tried to maximize beam bending and beam axial stiffness using IHR. Duvaut et al. [7] developed a method for determining the optimal direction and volume fraction of fibers at each point of a structure. The fiber orientation and the fiber volume fraction were assumed to be constant within each element of the model, but they varied from element to element.

For thin and large composite plates subject to in-plane compressive loads, buckling is the most critical failure mode. For this reason, design of composites for optimum bucking response drew attention of researchers [8–14]. Optimization of buckling response was first formulated by treating ply angle and thickness as design variables [10], which lead to a linear integer programming problem. However, imposing any constraint made the problem nonlinear. Branch and Bound algorithm was then applied to solve this problem [10–12]. Le Riche and Haftka [13] applied a genetic algorithm to maximize the buckling load and imposed constraints to avoid ply proximity and strain failure. Soremekun et al. [14] considered the same problem using a genetic algorithm with generalized elitist condition. The searched region was found to contain multiple global and near-global optima. However, the effectiveness of GA in locating global optimum designs in larger design spaces, i.e. for a higher number of design variables, has not been tested.

In the present study, we attempted to develop a procedure that can locate global optimum designs of composite laminates with minimum liability to buckling even in a very large design space. For this purpose, the simulated annealing (SA) was used as a suitable global search algorithm. SA is generally more reliable in finding the global optimum, i.e. the probability of locating the global optimum is high even with large numbers of design variables. Nevertheless, SA has not been used in previous studies for a similar problem. Its application even to optimization of homogeneous structures [15,16] is rare.

In this study, we adopted an improved version of SA called the direct search simulated annealing (DSA) developed by Ali et al. [17]. The algorithm was applied to problems with increasing difficulty. First, solution of a problem was attempted which was known to have multiple global optima. Then, the design space was considerably increased to see the effectiveness of the algorithm.

1.1. Problem statement

The composite panel under consideration is simply supported on four sides with a length of a and width of b (Fig. 1). The panel is subject to given in-plane compressive loads N_x and N_y in the x and y directions, respectively. The laminate is symmetric, balanced about the midplane and made of layers each of thickness t.

The laminate buckles into m and n half-waves in the x and y directions, respectively, when the loads reach the values $\lambda_b N_x$ and $\lambda_b N_y$. λ_b is the load amplitude, which is defined in terms of bending stiffnesses D_{ij} and loads as [18]

$$\lambda_b(m,n) = \pi^2\left[\frac{m^4 D_{11} + 2(D_{12}+2D_{66})(rmn)^2 + (rn)^4 D_{22}}{(am)^2 N_x + (ran)^2 N_y}\right] \tag{1}$$

where r is the plate aspect ratio, defined as the ratio of the length to the width, a/b. The critical buckling load factor λ_{cb}, which limits the maximum load that the laminate can sustain without buckling, is the smallest value

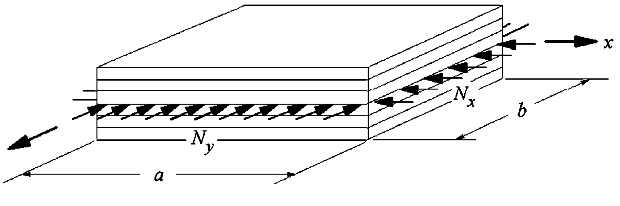

Fig. 1. A symmetric laminate under compressive biaxial loads.

of λ_b under any combination of pair (m,n), which should be greater than one to avoid immediate failure.

$$\lambda_{cb} = \min \lambda_b(m,n) \tag{2}$$

Taking $\{M,N\} = 2$ was shown to result in a good estimate of buckling load capacity. Accordingly, the smallest of $\lambda(1,1)$, $\lambda\,(1,2)$, $\lambda(2,1)$ and $\lambda(2,2)$ was taken as the critical buckling load [18].

The optimization problem that we consider is to find the optimum configuration of the composite having the maximum critical buckling load factor, λ_{cb}.

2. Simulated annealing algorithm

Kirkpatrick et al. [19] first proposed simulated annealing as a powerful stochastic search technique. The method gets its name from the physical process whereby the temperature of a solid is raised to a melting point, where the atoms can move freely, and then slowly cooled. The method attempts to model the behavior of the atoms in forming arrangements in solid material during annealing. Although the atoms move randomly, as their natural behavior they tend to form lower-energy configurations. However, this is a time driven process. When a material is crystallized from its liquid phase, it must be cooled slowly if it is to assume its highly ordered, lowest-energy, perfect crystalline state. At each temperature level during this annealing process, the material should reach equilibrium. As the temperature decreases, the arrangement of the atoms gets closer and closer to the lower energy state. This process continues until the temperature finally reaches freezing point.

There is an analogy between the physical annealing process and an optimization process. Different configurations of the problem correspond to different arrangements of the atoms. The cost of a configuration corresponds to the energy of the system. Optimal solution corresponds to the lowest energy state. Just in the same manner the atoms find their way to build a perfect crystal structure through random movements, the global optimum is reached through a search within randomly generated configurations.

In the SA algorithm, a random initial point is selected and systematically updated until a stopping criterion is satisfied. Updating is based on an iterative procedure. In each iteration, a random point is generated in the neighborhood of the current configuration. If the new point has a smaller value of cost function compared to that of the current record, the point is accepted. This point replaces the old one. On the other hand, if the new cost function has a larger value, the acceptability of the point is decided according to the probability of Boltzman distribution. The calculation of this probability depends on a temperature parameter, T, which is referred to as temperature, because it plays a similar role in the optimization process as the temperature in the physical annealing process. The temperature is kept constant for a number of trials and then reduced. The rate of reduction should be slow so as not to get trapped at a local minimum point. At initial stages of the algorithm (at high temperatures), the probability of accepting worse designs is higher but later on at low temperatures, this probability becomes smaller and smaller so that in the end the designs having higher cost are almost never accepted.

DSA, differs from SA basically in two aspects. Firstly, DSA uses a set of current configurations rather than just one current configuration. Secondly, it always retains the best configuration. In a way, this property imparts a sort of memory to the optimization procedure. If a newly generated configuration is accepted, it just replaces the worst configuration. DSA like SA requires random generation of configurations.

3. The optimization procedure

For the optimum design problem considered in this study, a configuration represents a composite laminate consisting of layers in which fibers are oriented in specific directions. The design variables are the orientation angles of the layers forming the composite laminate. By varying the fiber orientations, one may obtain a different configuration of the laminate having a different response. The thickness of each layer is the same and taken as constant.

3.1. The set of current configurations

DSA unlike ordinary SA starts with a set of N number of configurations, A, rather than starting with only one configuration. The number of these configurations depends on the dimension of the problem.

$$N = 7(n+1) \tag{3}$$

where n is the dimension of the problem, i.e. the number of design variables. Since the laminate is symmetric, and the orientation of the fiber in each ply is a design variable, n is given by

$$n = \frac{n_m}{2} \tag{4}$$

where n_m is the total number of the plies.

In the first step of the algorithm, an initial set of N current configurations are randomly generated within the solution domain, S. The cost function values of these N configurations are stored. The highest and lowest costs are denoted as f_h and f_ℓ.

3.2. Generation mechanisms for a new configuration

There are two different mechanisms for generating a new configuration. A configuration may be generated randomly with probability q, or it may be generated using $n + 1$ configurations selected from the set of current configurations (A) with probability $1 - q$. This is called controlled generation. Then,

$$\begin{matrix}\text{Generation}\\ \text{mechanism}\end{matrix} = \begin{cases}\text{Controlled generation} & \text{if } w \geqslant q\\ \text{Random generation} & \text{if } w < q\end{cases} \tag{5}$$

where w is a random number and q is a number selected by the designer to provide a bias for one of the mechanisms. For the problems considered in this study, 0.85 was observed to be an appropriate value for q in our test runs.

In random generation (RG), first a configuration is randomly selected from the set of current configurations, A, and then fiber orientations, θ_i, of its layers are randomly changed, resulting in a new configuration. In controlled generation (CG), a random selection of n configuration, $c_2, c_3, \ldots, c_{n+1}$ is first made from A, and then the fiber orientation angle, θ_i, of each ply is calculated using $c_1, c_2, c_3, \ldots, c_n$; c_1 being the best configuration in A, as follows

$$\theta_i = 2\overline{\theta}_i - \theta_i^{n+1} \tag{6}$$

where $\bar{\theta}_i$ and is the average of the fiber angles for the ith layers of $c_1, c_2, c_3, \ldots, c_n$.

$$\bar{\theta}_i = \frac{1}{n}\sum_{k=1}^{n}\theta_i^k \tag{7}$$

Since the fiber angles are constrained to have discrete values, the values of $\bar{\theta}_i$ are rounded to the closest acceptable fiber angle. If any of the angles falls outside the feasible region, S, i.e. being greater than 90° or less than 0°, the above process is repeated.

3.3. Acceptability

Acceptability of a newly generated trial configuration depends on the value of its cost, which is just negative of critical buckling load factor, λ_{cb}. If it is f_t, its acceptability, A_t, is calculated by

$$A_t = \begin{cases}1 & \text{if } f_t \leqslant f_h\\ \exp((f_h - f_t)/T_k) & \text{if } f_t > f_h\end{cases} \tag{8}$$

Here f_h is the highest cost in A. This means every new design having a cost lower than the cost of the worst design is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of A_t. If it is greater than a randomly generated number, P_r, the trial configuration is accepted, otherwise it is rejected.

If the trial design is accepted, it replaces the worst configuration. The best configuration, thus, always remains in A. In each iteration f_h and f_ℓ are updated. At high temperatures it is unlikely that A will form a dense cluster, which means current configurations scatter around within the solution domain. At low temperatures, because the likelihood of accepting a worse configuration is low, A would shrink to form a dense cluster.

3.4. Cooling schedule

The simulated annealing process consists of first "melting" the system being optimized at a high "temperature", T, then lowering the temperature slowly until the system "freezes" and no further changes occur. Here, temperature, T, has no physical meaning; it is just a control parameter. Melting corresponds to the stage where initial configurations are generated within the solution domain, S, without much regard to the cost. At high temperatures, (high values of T) the probability of acceptance is high as Eq. (8) implies. Accordingly, configurations that have even very high cost values may be accepted, just as in the physical annealing process, the atoms may form configurations having very high energy in the melting state. At low values of the temperature parameter, acceptability becomes low, and acceptance of worse configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements at low temperatures. The cooling schedule in SA controls the convergence of the algorithm to the global minimum just as the cooling scheme in the physical annealing process controls the final microstructure. Therefore, the performance of SA depends on the cooling schedule.

In a cooling schedule, first an initial value, T_0 should be specified for the temperature parameter. A scheme is then required for reducing T and for deciding how many trials are to be attempted at each value of T. Lastly, the final value of the temperature parameter should be specified.

Initial value of the temperature parameter: The value of the initial temperature parameter, T_k, should be large enough to allow virtually all trials to be accepted. This means that the initial acceptance ratio $\chi_0 = \chi(T_0)$ should be close to one. This provides complete melting at the initial stages of the optimization process. Otherwise, searched region will be small and then the algorithm may become trapped in a local minimum. In the physical analogy mentioned earlier, choosing high T_0 corresponds to heating up the solid until all particles are randomly arranged in the liquid phase so that atoms may freely arrange themselves.

Length of the Markow Chains: Iterations during which the value of the temperature parameter (T) is kept constant are called Markov chains (or inner loops). Ali et al. [17] adopted the following equation to decide on the length of a Markow chain (the number of trials (or iterations)) for the kth value of T:

$$L_k = L + L(1 - e^{f_\ell - f_h}) \tag{9}$$

Here

$$L = 10n \tag{10}$$

where n is the dimension of the problem given by Eq. (4). At high temperatures, when the current configurations form a sparse cluster, consequently $f_h - f_\ell$ is large, Markow chain length is close to $2L$. On the other hand, when they form a dense cluster at low temperatures, it approaches to L. During the execution of kth Markow chain with length L_k, if a configuration is generated having a cost less than f_ℓ, the best value in A, the current chain is stopped, and a new Markow chain begins. If a configuration better than the best configuration in A is not found, the complete chain of length L_k is executed.

The scheme for decreasing the temperature parameter: After the initial value of the temperature parameter, T_0, is determined, a decrement rule must be established to find the subsequent values of the temperature parameter. The probability of reaching the global optimum solution depends on how fast the value of the temperature parameter is lowered. If the cooling rate is fast, the optimization process will probably end up with one of the higher-cost local minima. If the cooling rate is slow, the global optimum solution may be reached, but, the optimization process will take excessively long time. Therefore, the choice of the cooling schedule determines the effectiveness of the algorithm.

In DSA, a temperature scale factor (α_{k+1}), is specified to calculate the value of the temperature parameter in the next Markow chain, T_{k+1}.

$$T_{k+1} = \alpha_{k+1} T_k \tag{11}$$

where T_k is the value of the current temperature. A value for $\alpha_{k+1} \in [\alpha_{\min}, \alpha_{\max}]$ is calculated using the following equation [17]:

$$\alpha_{k+1} = \begin{cases} \alpha_{\max} & \text{if } L_k > L'_k \\ \alpha_k - (\alpha_k - \alpha_{\min})(1 - L'_{k-1}/L_k) & \text{else if } L_k > L'_{k-1} \\ \alpha_{\max} - (\alpha_{\max} - \alpha_k)(L_k/L'_{k-1}) & \text{else } L_k \leqslant L'_{k-1} \end{cases} \tag{12}$$

where L'_k is the actual number of trials executed in the kth Markow chain. If no configuration is found in the kth Markow chain (inner loop) that is better than the best current configuration L'_k is set to the value of L_k. If a configuration which is better than all is found, the inner loop is terminated, and L'_k is set to the number of iterations actually executed in this loop. In this study, we chose $\alpha_{\max} = 0.997$ and $\alpha_{\min} = 0.97$ as appropriate values, which provided a compromise between the two effectiveness criteria, i.e. short computational time and high reliability of locating global minimum. Initial value of α is the average of $\alpha_{\max}$ and $\alpha_{\min}$.

Stopping criterion: The optimization process is terminated when two conditions are satisfied. Firstly, the current temperature parameter should be small, in other words it should be close to the "freezing" (zero) temperature, i.e. no further improvement may occur, and secondly, the current configurations, A, should form a dense cluster. Accordingly, the stopping criterion can be expressed as

$$T_k < \varepsilon_1 \quad \text{and} \quad f_h - f_\ell < \varepsilon_2 \tag{13}$$

where ε_1 and ε_2 are small numbers. For the optimization problems that were considered, it was more than sufficient to take $\varepsilon_1 = \varepsilon_2 = 0.008$ for a good estimate of the global minimum.

Fig. 2 describes the procedure followed in this study for the optimization problem. Here k and j show the inner loop number and the number of generated configurations in the current inner loop, respectively.

4. Results

For the design optimization problem, a graphite epoxy plate was chosen with the elastic properties of $E_{11} = 127.59$ GPa, $E_{22} = 13.03$ GPa, $G_{12} = 6.41$ GPa, and $\nu_{12} = 0.28$. The dimensions of the composite panel were $a = 50.8$ cm, $b = 25.4$ cm, and $t = 0.127$.

Results were obtained for various sets of fiber orientations and load cases. First, the problem studied by Le Riche and Haftka [13] and Soremekun et al. [14] was considered. This problem was known to have multiple-global optima as discussed in these studies. The composite panel had totally 64 layers having the same thickness, and comprised 2-ply-stacks with possible fiber orientations of 0_2, ±45 and 90_2. The laminate was symmetric and balanced. Accordingly, 16 fiber orientations as design variables, which may take three different values, were sufficient to describe the entire laminate.

The initial temperature was set to 500. The optimization process was repeated 50 times in order to see the effectiveness and reliability of the algorithm. The design space consisted of 43.05×10^6 different configurations and on the average 15,000 iterations were carried out. The results are shown in Table 1. In the previous studies, six global optima were found. In this study, two more global optima were located. Table 1 also lists the number of runs within these 50 runs in which a given number of these optima was found, and the average number of the optima found per run. In none of the runs, all of the eight of these global optima were found. On the other hand, none of the runs failed to find at least one global optimum. There was only a single run in which six of these were located. It should be noted that, the algorithm was not revised for locating multiple global optimums.

In order to see the effectiveness of the algorithm, design space was enlarged by allowing fiber orientation to change in every layer, (thus, the number of design

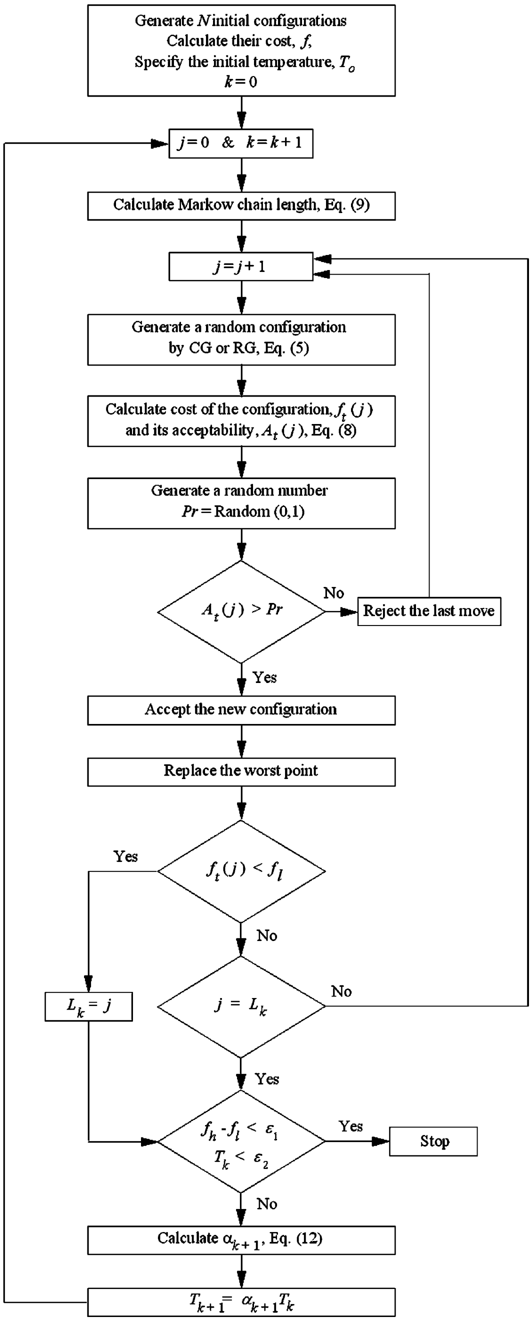

Fig. 2. The optimization procedure.

variables became 32), and by increasing the number of possible fiber angles. First, as possible fiber directions 0°, 30°, 60°, and 90° were chosen, then 0°, 15°, 30°, 45°, 60°, 75°, 90°, and then 0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, and 90°. These fiber orientations yield 1.845×10^{19}, 1.1×10^{27}, 1.0×10^{32} different designs, respectively. As the design space was enlarged, initial temperature was increased to 2000, 2500, and 3000 for increments of 30°, 15°, 10° respectively, in order to be able to search a larger region. As seen in Table 2, the results show that it is possible to find better designs having a larger buckling load capacity if more fiber orientations are allowed. Besides the multiple global minimum, the algorithm was able to find numerous near optimum designs, e.g. $[90/60/90_4/60_2/90/60/90_3/60_3/90_2/60_3/90_2/60_2/90_2/60/90//60/90]_s$ and $[90_2/60_2/90_3/60/90/60_2/90_4/60_2/90/60_2/90/60_3/90\ /60_4/90_3]_s$ having buckling load factors 4080.06 and 4080.04, respectively.

Table 1
The global optimal designs for $N_x/N_y = 1$, $a/b = 2$, 64 plies with possible fiber angles of 0_2, ± 45, 90_2

Design	Buckling load factor
$[90_{10}/\pm 45_2/90_2/\pm 45_3/90_2/\pm 45_4]_s$	3973.01
$[90_8/\pm 45/90_2/\pm 45/90_2/\pm 45/90_2/\pm 45_6]_s$	3973.01
$[90_{10}/\pm 45/90_2/\pm 45_7/90_2/\pm 45]_s$	3973.01
$[\pm 45/90_{10}/\pm 45/90_8/\pm 45/90_8]_s$	3973.01
$[\pm 45/90_8/\pm 45/90_{18}/\pm 45]_s$	3973.01
$[90_4/\pm 45_2/90_{16}/\pm 45/90_6]_s$	3973.01
$[90_2/\pm 45/90_6/\pm 45/90_8/\pm 45/90_{10}]_s$	3973.01
$[90_4/\pm 45_2/90_{16}/\pm 45/90_6]_s$	3973.01

# of optima found over 50 runs									Average # of optima/run
0	1	2	3	4	5	6	7	8	
0	3	10	21	11	4	1	0	0	3.12

Tables 3–6 show the results for various N_x/N_y. The algorithm again found multiple global designs. As seen in the tables the fibers tend to take more acute angles with respect to x axis when the load ratio increases, i.e. the load in the x- direction becomes larger than the load in the y-direction. Also as the load ratio decreases the fiber orientations approach to 90°. Finally, when the load ratio is 0.25 an optimum configuration is obtained in which all the fibers are orientated at 90°. So it is reasonable to conclude that decreasing the load ratio further will not result in a different configuration. Considering the overall data, one may conclude that as the number of possible fiber orientations increases, the algorithm may find designs having a larger load capacity.

4.1. Reliability of the algorithm

The results were obtained over 40 runs in order to observe the reliability of the algorithm. Table 7 lists the number of the runs in which a specific number of global optima was found. The percentage of the optima found in the 40-runs sample is also given. As the design space enlarges, average number of optima per run decreases for all load cases. Also, the results seem to be consistent when the same optimization variables were used for different load conditions. The maximum number of optima found in a single run was four, which is the case for a load ratio of one and a decrement of 30°. As the design space enlarges, the reliability of the algorithm becomes lower. In order to obtain a higher reliability, one need to lower the temperature parameter, T, more slowly; this also leads to longer computational times.

Table 2
The global optimum configurations for $N_x/N_y = 1$, $a/b = 2$, 64 plies

Possible fiber orientations and one of the global designs	No. of global optimums	Buckling load factor
Fiber directions: 0–30–60–90 $[90_5/60/90/60/90/60/90/60_6/90_2/60_8/90/60_2/90/60]_s$	21	4080.08
Fiber directions: 0–15–30–45–60–75–90 $[75_3/60/75_3/60/75_{10}/60/75_8/60/75_2/60_2]_s$	10	4114.81
Fiber directions: 0–10–20–30–40–50–60–70–80–90 $[70/80/70_4/80/70_2/80/70_3/80/70_4/80_4/70_2/80_2/70/80/70/90/70/60]_s$	1	4123.28

Table 3
The global optimum configurations for $N_x/N_y = 2$, $a/b = 2$, 64 plies

Possible fiber orientations and one of the global designs	No. of global optimums	Buckling load factor
Fiber directions: 0–30–60–90 $[90/60_{10}/90/60_6/90/60_{11}/90/60]_s$	12	6379.35
Fiber directions: 0–15–30–45–60–75–90 $[60_4/75/60_4/75/60_4/75_4/60_{13}/75]_s$	12	6392.73
Fiber directions: 0–10–20–30–40–50–60–70–80–90 $[60_9/70_4/60_2/70_8/60/70/60/70_2/60/70/60/70]_s$	10	6403.64

Table 4
The global optimum configurations for $N_x/N_y = 4$, $a/b = 2$, 64 plies

Possible fiber orientations and one of the global designs	No. of global optimums	Buckling load factor
Fiber directions: 0–30–60–90 $[60_{32}]_s$	1	8026.83
Fiber directions: 0–15–30–45–60–75–90 $[45/60_4/45/60/45_4/60_3/45_7/60_7/45/60/45_2]_s$	14	8440.27
Fiber directions: 0–10–20–30–40–50–60–70–80–90 $[60/50_5/60/50/60_2/50_6/60_2/50_3/60/50/60/50_4/60/50_3]_s$	5	8543.46

Table 5
The global optimum configurations for $N_x/N_y = 1/2$, $a/b = 2$, 64 plies

Possible fiber orientations and one of the global designs	No. of global optimums	Buckling load factor
Fiber directions: 0–30–60–90 $[90_{15}/60/90/60/90_2/60_3/90_2/60/90_2/60_4]_s$	26	4741.92
Fiber directions: 0–15–30–45–60–75–90 $[90_8/75/90/75/90/75/90_2/75_5/90/75_2/90/75_8]_s$	17	4754.20
Fiber directions: 0–10–20–30–40–50–60–70–80–90 $[90_2/80_2/90/80/90_2/80_4/90/80_2/90/80_3/90/80_5/90/80/90/80/90_2/80]_s$	3	4756.28

Table 6
The global optimum configurations for $N_x/N_y = 1/4$, $a/b = 2$, 64 plies

Possible fiber orientations and one of the global designs	No. of global optimums	Buckling load factor
Fiber directions: 0–30–60–90 $[90_{32}]_s$	1	5074.79
Fiber directions: 0–15–30–45–60–75–90 $[90_{32}]_s$	1	5074.79
Fiber directions: 0–10–20–30–40–50–60–70–80–90 $[90_{32}]_s$	1	5074.79

5. Conclusions

In this study, the direct simulated annealing algorithm was adopted to maximize the load that a composite laminate can sustain without buckling. Fiber orientations of individual layers were taken as design variables. First, a problem that was known to have multiple-global optima was considered. After observing the success of the algorithm in locating all of the global optimum designs, the design space was enlarged by increasing the number of possible fiber angles. As expected, the algorithm was able to find designs having the same thickness but larger buckling load capacity. For different

Table 7
Reliability table for buckling load problem

Loading condition	Optimization variables	# of optima found over 40 runs					Ave. # of optima/run
		0	1	2	3	4	
$(N_x/N_y) = 1$	0–30–60–90	12	18	4	5	1	1.1
	0–15–30–45–60–75–90	20	15	3	2	0	0.675
	0–10–20–30–40–50–60–70–80–90	22	18	–	–	–	0.45
$(N_x/N_y) = 2$	0–30–60–90	6	26	7	1	0	1.075
	0–15–30–45–60–75–90	19	16	3	2	0	0.7
	0–10–20–30–40–50–60–70–80–90	16	24	0	0	0	0.6
$(N_x/N_y) = 4$	0–30–60–90	3	37	–	–	–	0.925
	0–15–30–45–60–75–90	16	18	6	0	0	0.75
	0–10–20–30–40–50–60–70–80–90	20	17	3	0	0	0.575
$(N_x/N_y) = 1/2$	0–30–60–90	8	20	9	3	0	1.175
	0–15–30–45–60–75–90	14	21	5	0	0	0.775
	0–10–20–30–40–50–60–70–80–90	23	17	0	–	–	0.425
$(N_x/N_y) = 1/4$	0–30–60–90	0	40	–	–	–	1
	0–15–30–45–60–75–90	16	24	–	–	–	0.6
	0–10–20–30–40–50–60–70–80–90	18	22	–	–	–	0.55

load ratios, reliability of the algorithm was investigated. For problems in which the design space is very large, the need for slower "cooling" was indicated.

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