



Optimum design of composite laminates for minimum thickness

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ABSTRACT

In this study, an optimization procedure is proposed to minimize thickness (or weight) of laminated composite plates subject to in-plane loading. Fiber orientation angles and layer thickness are chosen as design variables. Direct search simulated annealing (DSA), which is a reliable global search algorithm, is used to search the optimal design. Static failure criteria are used to determine whether load bearing capacity is exceeded for a configuration generated during the optimization process. In order to avoid spurious optimal designs, both the Tsai–Wu and the maximum stress criteria are employed to check static failure. Numerical results are obtained and presented for different loading cases.

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1. Introduction

Fiber-reinforced composite materials are demanded by the industry because of their high specific stiffness/strength especially for applications where weight reduction is critical. By using composites, weight of a structure can be reduced significantly. Further reduction is also possible by optimizing the material system itself such as fiber orientations, ply thickness, stacking sequence, etc. Many researchers attempted to make a better use of material either by minimizing the laminate thickness [1–23], thus reducing the weight, or by maximizing static strength of composite laminates for a given thickness [10,16,24–36].

In some of these studies, layer thickness [1–4,7,9–12,15,17,24] and ply angles [3,4,7,9–11,13,15,17,25–29,31,36] were considered to be continuous design variables. However in practice, composite laminates are fabricated using prepregs with a specific thickness. Besides, fiber orientations are chosen from a finite set of angles during the design process because of the difficulty of exactly orienting fibers along a given direction. If layer thickness and fiber orientation angles are taken as continuous variables in an optimization process, the optimum values should be converted to the nearest discrete manufacturable values. In that case, the resulting design may not be optimal; besides constraints may be violated. Due to these manufacturing constraints, the design variables for a fiber angle or layer thickness should take only discrete values. As opposed to a zero order search algorithm, a gradient based optimization procedure may fail to cope with the discrete nature of such problems. Moreover, in typical structural optimization problems, there may be many locally optimum configurations. With a large number of design variables, the number of local minima may increase dramatically.

Composite structures having additional design variables such as fiber orientation and layer thickness pose great difficulty in locating the globally optimum designs. This is especially true for deterministic local search algorithms, which usually stick to one of the local minima. On the other hand, stochastic optimization methods are more suitable tools. Among their advantages, they are not sensitive to starting point, they can search a large design space, and they can escape local optimum points because they allow occasional uphill moves. In this study, a stochastic search algorithm called “direct search simulated annealing” (DSA), proposed by Ali et al. [37], was adopted. This is an improved variant of the simulated annealing (SA) algorithm [38], which is known as one of the most reliable algorithms in locating globally optimum configurations.

As another difficulty in the design optimization of composite structures, use of the Tsai–Wu criterion, which is one of the most reliable static failure criteria, leads to false optimum design for some loading cases due to the particular feature of its failure envelope. In some studies [27,28], in order to avoid this condition, the compressive strength of the material was taken the same as its tensile strength. In a design process, assuming the material to be weaker than it actually is leads to overly conservative designs. For this reason, this assumption is against the purpose of design optimization. In order to overcome this difficulty, we employed the Tsai–Wu criterion together with the maximum stress criterion and observed satisfaction of both of them.

2. Problem formulation

2.1. Problem statement

The structure to be optimized is a symmetric 2-D multilayered structure reinforced by continuous fibers subject to in-plane

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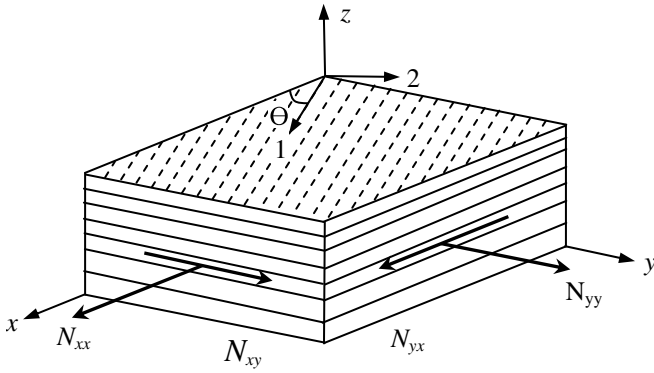


Fig. 1. A scheme of the composite structure considered in this study.

normal and shear loading as shown in Fig. 1. Accordingly no bending and twisting moments are considered in the analysis of its mechanical behavior.

The laminate consists of plies having the same thickness. The objective is to find the optimum design of the laminate to attain the minimum possible laminate thickness with the condition that it does not fail.

Minimize t (1)

where t is the thickness of the laminate.

The number of distinct fiber orientation angles, m , is given. The orientation angles, θ_k , and how many plies, n_k , are oriented along each angle are to be determined in the design process. Accordingly, the number of design variables is $2m$. The laminate thickness can be expressed as

$$t = 2t_0 \sum_{k=1}^m n_k \quad (2)$$

where t_0 is the thickness of an individual ply and n_k is the number of plies with fiber angle θ_k . The factor “2” appears because of the symmetry condition for the laminate with respect to its middle plane. Because the plies are made of the same material, minimizing thickness leads to the same optimum configuration as the minimization of weight.

The orientation angles take discrete values; they are chosen from a given set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, 1°, 0.5° or even smaller.

2.2. Analysis of a laminated composite plate

The classical lamination theory is used to analyze the mechanical behavior of the composite laminate. We assume that plane stress condition is valid for each ply. Accordingly, out-of-plane stress components are taken as zero. With respect to the coordinate system shown in Fig. 1, the in-plane stress components are related to the strain components as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_k \quad (3)$$

where k is the lamina number counted from the bottom, \bar{Q}_{ij} are the off-axis stiffness components, which can be expressed in terms of principal stiffness components, Q_{ij} , using the tensor transformation rules [39] as

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{12} - Q_{22} + 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66} (\sin^4 \theta + \cos^4 \theta) \end{aligned} \quad (4)$$

The principal stiffness terms, Q_{ij} , are related to elastic properties of the material along the principal directions, $E_1, E_2, G_{12}, \nu_{12}$, and ν_{21} [39]. Because, the laminate is only subject to in-plane loads and it is symmetric, curvature terms become zero. Then, it remains flat; therefore strain components defined with respect to x - y - z coordinate system are the same for each ply regardless of the fiber orientation ($\{\varepsilon\}_k = \{\varepsilon\}$). For the same reason, the mechanical response of the laminate is independent of the stacking sequence.

Stress resultants, or forces per unit length of the cross section, are obtained by through-the-thickness integration of the stresses in each ply.

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz = 2 \sum_{k=1}^m n_k t_0 \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k \quad (5)$$

Here m is the number of distinct laminae in one of the symmetric portions above or below the mid-plane, n_k is the number of plies in the k th lamina. Here, lamina is meant to be a group of plies with the same orientation angle. Substituting the stress-strain relation given by Eq. (3) into Eq. (5), we get

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix} \quad (6)$$

where A_{ij} , components of extensional stiffness matrix, are given by

$$A_{ij} = 2 \sum_{k=1}^m n_k t_0 (\bar{Q}_{ij})_k \quad (7)$$

Given the loading, N_{xx}, N_{yy} , and N_{xy} , one may obtain off-axis strain components $\varepsilon_{xx}, \varepsilon_{yy}$, and γ_{xy} , using Eq. (6), and then off-axis stress components in each ply $\sigma_{xx}^k, \sigma_{yy}^k$, and τ_{xy}^k using Eq. (3). Principal stress components can be obtained using the following transformation:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} \cos^2 \theta_k & \sin^2 \theta_k & 2 \cos \theta_k \sin \theta_k \\ \sin^2 \theta_k & \cos^2 \theta_k & -2 \cos \theta_k \sin \theta_k \\ -\cos \theta_k \sin \theta_k & \cos \theta_k \sin \theta_k & \cos^2 \theta_k - \sin^2 \theta_k \end{bmatrix}_k \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k \quad (8)$$

2.3. Static failure criteria

Weight minimization of composite structures necessarily involves strength constraints, because decreasing number of load carrying plies eventually leads to failure. The structure must be able to withstand the imposed loads without suffering any failure. In this study, only the static failure modes are assumed to be critical for the laminates. The other failure modes, low stiffness, buckling, delamination, etc. are assumed to be not critical.

In order to check the feasibility of a configuration generated by the search algorithm during an optimization process, one needs to use reliable failure criteria. A common approach is to use a limit theory such as the maximum stress criterion. According to this criterion, failure is predicted whenever one of the principal stress

components exceeds its corresponding strength. The failure envelope for a ply under in-plane normal and shear stresses is then defined by

$$\sigma_{11} < X_t \text{ and } \sigma_{11} > X_c \text{ and } \sigma_{22} < Y_t \text{ and } \sigma_{22} > Y_c \text{ and } |\tau_{12}| < S \tag{9}$$

where “X” and “Y” denote the strength along the fiber direction and transverse to it, respectively; the subscripts “t” and “c” signify the tensile and compressive strengths; S, on the other hand, is the ultimate in-plane shear strength of a laminate under pure shear loading. Adopting the first-ply-failure criterion, the whole laminate is assumed to have failed, if one of these inequalities is not satisfied for any one of the laminae. Once the stress state in the principal coordinates (σ_{11} , σ_{22} , and τ_{12}) for each lamina is determined, it is straightforward to apply this failure criterion.

Although the maximum stress criterion is easy to apply, it does not account for the interaction between the effects of different stress components. Fig. 2 shows the safety factor calculated using this criterion for a laminate subject to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ . Two different laminate lay-up configurations were considered. One is a balanced and symmetric laminate, $[\theta_{25}/-\theta_{25}]_s$, the other is a unidirectional laminate, $[\theta_{51}]_s$. One may observe sudden changes in the trend line as one of the inequalities in Eq. (9) becomes inactive while one of the others becomes active due to a small change in θ . This does not conform to the empirically observed trends. The reason for this lies in the incapability of the maximum stress criterion to reflect the interactive effects. We may also observe that for the balanced laminate, $[\theta_{25}/-\theta_{25}]_s$, the criterion correctly predicts that the laminate is strongest for $\theta = 0^\circ$, in which fibers are oriented along the loading direction. The safety factor for this case is less than one; but is the highest of all. However, for the unidirectional laminate, $[\theta_{51}]_s$, the criterion falsely predicts the highest safety factor for $\theta = 5^\circ$. This means that an optimization process in which failure is assessed based on the maximum stress criterion may stick to a spurious optimum design for an unbalanced laminate. Although, coupling between normal and shear strains occurs in an unbalanced laminate, for many applications, this may be tolerated. A general design optimization procedure should then be able to optimize unbalanced laminates. Accordingly, failure analysis should not solely be based on this criterion.

The Tsai–Wu failure criterion is one of the most reliable static failure criteria as it provides a simple analytical expression taking into account the competing interactive effects among the stress components. Its general form for orthotropic materials under plane stress assumption is expressed as [39,40]

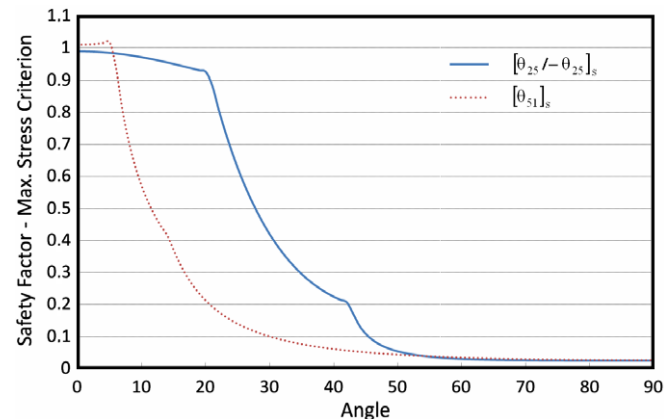


Fig. 2. Safety factor calculated for a laminate subject to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ , using the maximum stress criterion.

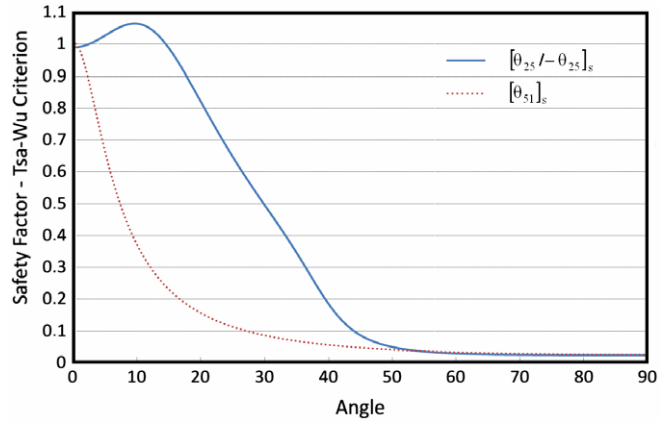


Fig. 3. Safety factor calculated for a laminate subject to uniaxial loading (only $N_{xx} \neq 0$) for various fiber orientation angles, θ , using the Tsai–Wu criterion.

$$\frac{\sigma_{11}^2}{X_t|X_c|} + \frac{\sigma_{22}^2}{Y_t|Y_c|} + \frac{\tau_{12}^2}{S^2} - \frac{\sigma_{11}\sigma_{22}}{\sqrt{X_tX_cY_tY_c}} + \left(\frac{1}{X_t} - \frac{1}{|X_c|}\right)\sigma_{11} + \left(\frac{1}{Y_t} - \frac{1}{|Y_c|}\right)\sigma_{22} < 1 \tag{10}$$

Here the coefficient in front of $\sigma_{11}\sigma_{22}$, which explains the interaction among normal stress components, is expressed in terms of the available uniaxial strengths as the other coefficients. Since in this form it does not require data obtained through biaxial stress tests, the Tsai–Wu criterion is as easy to apply as the maximum stress criterion. Fig. 3 shows the safety factor calculated using the Tsai–Wu criterion. For the unidirectional laminate, $[\theta_{51}]_s$, the criterion correctly predicts that the laminate is strongest for $\theta = 0^\circ$. The safety factor quickly decays with increasing θ . However, for the balanced laminate, $[\theta_{25}/-\theta_{25}]_s$, the criterion falsely estimates the highest safety factor as 1.065 at $\theta = 10^\circ$. Actually, the laminate is expected to fail. One may conclude that the Tsai–Wu criterion will also lead to false optimum designs in an optimization process.

Considering the predictions of these two failure criteria for the two different laminate designs, each criterion seems to compensate the deficiencies of the other. By enforcing the satisfaction of both criteria, one may find the optimum design in both cases. In this study, both the maximum stress and the Tsai–Wu criteria are used to assess the load bearing capacity of a composite laminate with the hope that false optimum designs will be avoided for any laminate configuration.

3. Methodology

3.1. Formulation of the objective function

Failure of any ply signals inception of failure of the whole structure, even though its ultimate load bearing capacity may not be exceeded. For this reason, this is considered as a design limit. Accordingly, the first-ply failure approach is adopted in the design optimization and safety of each lamina in a laminate design generated during the optimization process is checked using the Tsai–Wu and maximum stress failure criteria. Failure is predicted if one of the inequalities in Eqs. (9) and (10) is not satisfied for one of the laminae. If a configuration generated during the optimization procedure leads to failure according to the failure criteria, a penalty value is calculated and added to the cost function. The overall cost function may then be expressed as

$$F = 2t_0 \sum_{k=1}^m n_k + w_1 P_{MS} + w_2 P_{TW} - w_1 SF_{MS} - w_2 SF_{TW} \tag{11}$$

where the first term represents the total thickness of the composite structure as given in Eq. (2); n_k is the number of plies in the k th lamina, in which the orientation angle is θ_k ; m is the total number of distinct laminae; the second and third terms represent the penalty values introduced to increase the value of the objective function for designs for which failure is predicted and thus to restrict the search to the feasible design space; P_{MS} and P_{TW} are penalty values calculated based on the maximum stress criterion and the Tsai–Wu criterion, respectively. SF_{MS} and SF_{TW} are equal to the safety factors according to the maximum stress and Tsai–Wu criteria, respectively, if they are greater than 1.0, otherwise these terms are equal to zero; w_i are suitable coefficients. The reason that the objective is reduced for safe designs is that there may be many feasible designs with the same minimum thickness. Of these designs, the optimum was defined as the one with the largest failure load. Accordingly, the objective function was linearly reduced in proportion to the failure margin as in Ref. [5]. Similarly in another study [21], the margins to initial failure were maximized with the minimum feasible number of laminae.

The safety factor of the laminate according to the maximum stress criterion, SF_{MS} , is calculated as follows: First, the principal stresses (σ_{11}^k , σ_{22}^k , and τ_{12}^k) in each lamina are determined; the safety factors for each failure mode are calculated; then the minimum of them is denoted as the safety factor of the lamina, SF_{MS}^k .

$$SF_{MS}^k = \min \left\{ \begin{array}{l} SF_X^k = \begin{cases} X_t/\sigma_{11} & \text{if } \sigma_{11} > 0 \\ X_c/\sigma_{11} & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} Y_t/\sigma_{22} & \text{if } \sigma_{22} > 0 \\ Y_c/\sigma_{22} & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = S/|\tau_{12}| \end{array} \right. \quad (12)$$

Then, the minimum of SF_{MS}^k is chosen among the safety factors of m number of different laminae as SF_{MS} .

$$SF_{MS} = \min \text{ of } SF_{MS}^k \quad \text{for } k = 1, 2, \dots, m - 1, m \quad (13)$$

The penalty value due to the violation of the maximum stress criterion is calculated as follows: If a principal stress component exceeds its respective strength, a penalty value is calculated.

$$P_X^k = \begin{cases} 0 & \text{if } SF_X^k \geq 1.0 \\ (1/SF_X^k) - 1 & \text{if } SF_X^k < 1.0 \end{cases} \quad (14)$$

$$P_Y^k = \begin{cases} 0 & \text{if } SF_Y^k \geq 1.0 \\ (1/SF_Y^k) - 1 & \text{if } SF_Y^k < 1.0 \end{cases}$$

$$P_S^k = \begin{cases} 0 & \text{if } SF_S^k \geq 1.0 \\ (1/SF_S^k) - 1 & \text{if } SF_S^k < 1.0 \end{cases}$$

The total penalty value for the laminate due to the violation of the maximum stress criterion is then calculated by summing up the penalty values calculated for each lamina.

$$P_{MS} = \sum_{k=1}^m P_X^k + P_Y^k + P_S^k \quad (15)$$

The safety factor for the k th lamina, SF_{TW}^k , according to the Tsai–Wu criterion is defined as the multiplier of the stress components at lamina k , σ_{ij}^k , that makes the right hand side of Eq. (10) equal to 1.0. Eq. (10) then becomes

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0 \quad (16)$$

where

$$a = \frac{(\sigma_{11}^k)^2}{X_t|X_c|} + \frac{(\sigma_{22}^k)^2}{Y_t|Y_c|} + \frac{(\tau_{12}^k)^2}{S^2} - \frac{(\sigma_{11}^k)(\sigma_{22}^k)}{\sqrt{X_tX_cY_tY_c}} \quad (17)$$

$$b = \left(\frac{1}{X_t} - \frac{1}{|X_c|} \right) \sigma_{11}^k + \left(\frac{1}{Y_t} - \frac{1}{|Y_c|} \right) \sigma_{22}^k \quad (18)$$

The root of Eq. (16) gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

$$SF_{TW}^k = \left| \frac{-b + \sqrt{b^2 + 4a}}{2a} \right| \quad (19)$$

Then, the minimum of SF_{TW}^k is chosen as the safety factor of the laminate

$$SF_{TW} = \min \text{ of } SF_{TW}^k \quad \text{for } k = 1, 2, \dots, m - 1, m \quad (20)$$

A penalty value is calculated and added to the objective function, if the Tsai–Wu criterion is violated at a lamina.

$$P_{TW}^k = \begin{cases} 0 & \text{if } SF_{TW}^k \geq 1.0 \\ (1/SF_{TW}^k) - 1 & \text{if } SF_{TW}^k < 1.0 \end{cases} \quad (21)$$

The total penalty value for the laminate due to the violation of the Tsai–Wu criterion is then found by summing up the penalty values calculated for each lamina.

$$P_{TW} = \sum_{k=1}^m P_{TW}^k \quad (22)$$

3.2. Optimization procedure

In this study, a variant of simulated annealing (SA) algorithm called “direct search simulated annealing” (DSA) [37] was used to minimize the thickness of laminated composite structures subject to in-plane loading. The application of DSA search algorithm to optimization of composite materials was explained in a previous study [41]. In this study, a number of improvements were introduced to increase the reliability of the algorithm.

In DSA unlike ordinary SA, a set of current configurations rather than a single current configuration is maintained during the optimization process. Accordingly, unlike the standard SA algorithm where only the neighborhood of a single point is searched, DSA searches the neighborhood of all the current points in the set. At the start of the optimization process, N number of initial configurations are randomly created within the design domain by randomly selecting values for the design variables. N is equal to $7(2m + 1)$, where $2m$ is the number of design variables as mentioned before. The design variables are the number of plies in the k th lamina, n_k , and the orientation angle of the fibers in these plies, θ_k . A number among $(0, 1, 2, \dots, 20)$ is randomly chosen for n_k , and among $(-90, -90 + \phi, \dots, -2\phi, -\phi, 0, \phi, 2\phi, \dots, 90 - \phi, 90)$ for θ_k . Here ϕ is the interval between consecutive angles, which may be $0.5^\circ, 1^\circ, 5^\circ, 10^\circ, 15^\circ, 30^\circ, 45^\circ$. If zero ply number is chosen, this means that no material exists for the respective lamina and this lamina does not contribute to the load carrying capacity of the laminate. After the initial laminate configurations are randomly chosen, their objective functions are calculated. DSA like SA requires random generation of a new configuration in each iteration. A configuration in the neighborhood of one of the current configurations is randomly generated as follows: First, one of the current configurations is randomly chosen. Then, random differences are introduced to the ply numbers and fiber angles.

$$\begin{aligned} n'_k &= n_k + r_1 \Delta n_{\max} \\ \theta'_k &= \theta_k + r_2 \Delta \theta_{\max} \end{aligned} \quad (23)$$

Here n_k and θ_k are the ply number and fiber angle in the k th lamina of the randomly chosen current configuration; n'_k and θ'_k are the ply number and fiber angle of the newly generated configuration; r_i are randomly chosen real numbers within $[-1, 1]$; Δn_{\max} and $\Delta \theta_{\max}$ are the maximum variations that may be introduced to n_k and θ_k to generate a new lay-up configuration. The lower limit for n'_k is zero; if n'_k

is negative, a new random number, r_1 , is generated. There is no upper limit for n'_k . The lower and upper limits for θ'_k are -90° and 90° , respectively. If a number greater than 90 is generated for θ'_k , 180 is subtracted from this number. If it is less than -90 , 180 is added. Acceptability of a newly generated trial configuration, A_t , depends on the value of its cost, f_t , which is calculated by

$$A_t = \begin{cases} 1 & \text{if } f_t \leq f_h \\ \exp((f_h - f_t)/T_j) & \text{if } f_t > f_h \end{cases} \quad (24)$$

Here f_h is the highest cost in the current set. This means every new design having a cost lower than the cost of the worst design is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of A_t . If it is greater than a randomly generated number, P_r , the trial configuration is accepted, otherwise it is rejected. If the trial design is accepted, it replaces the worst configuration. Iterations during which the value of the temperature (or control) parameter, T_j , is kept constant are called j th Markov chain (or inner loop). After a certain number of iterations, the temperature parameter, T , is reduced, a new inner loop begins. As Eq. (24) implies, when T is decreased, the probability that a worse configuration is accepted becomes lower. At low values of temperature parameter, acceptability becomes low; thus, acceptance of worse configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements at low temperatures.

In order to find the globally optimal design, one should be able to search a large solution domain. For this reason, instead of giving small perturbations to the current configuration to obtain a new configuration in its near neighborhood, one should allow a large variance in the current configurations. For this reason, the magnitudes of Δn_{\max} and $\Delta \theta_{\max}$ were taken as 15 and 50, respectively. This means that the neighborhood of a current configuration where a new configuration is generated is initially quite large. This can also be considered as a logical consequence of simulating the physical annealing process, where mobility of atoms is large at high temperatures, and thus the probability that atoms may form a quite different configuration is high. Also, as in the physical process, where mobility of atoms decreases as the temperature is lowered, variations in n_k and θ_k are also reduced as the temperature parameter is decreased; but the reduction scheme does not directly depend on temperature. The configuration that is worse than all current configurations except the worst one is defined as the worse configuration, and if no improvement is obtained in the worse configuration during a Markov chain, Δn_{\max} and $\Delta \theta_{\max}$ are reduced. For other details regarding the optimization procedure, one may refer to Ref. [41].

4. Numerical results and discussion

Two graphite/epoxy materials were considered in the lay-up sequence optimization. One is T300/5308 with the material properties of $E_{11} = 40.91$ GPa, $E_{22} = 9.88$ GPa, $G_{12} = 2.84$ GPa, $\nu_{12} = 0.292$ GPa, $X_t = 779$ MPa, $X_c = -1134$ MPa, $Y_t = 19$ MPa, $Y_c = -131$ MPa, $S = 75$ MPa. The other is T300/5208 with of $E_{11} = 181$ GPa, $E_{22} = 10.3$ GPa, $G_{12} = 7.17$ GPa, $\nu_{12} = 0.28$ GPa, $X_t = 1500$ MPa, $X_c = -1500$ MPa, $Y_t = 40$ MPa, $Y_c = -246$ MPa, $S = 68$ MPa.

As discussed before, relying on just one failure criterion may lead to false optimal designs. Use of a particular failure criterion will have impact on the safety and optimality of the resulting laminate design. Table 1 shows the dependence of optimal designs obtained by applying the aforementioned optimization procedure on the chosen failure criterion. The loading is uniaxial ($N_{xx} = 100 \times 10^6$ N/m) and two distinct fiber orientations are permitted. The interval between the angles is chosen to be 1° . If only the Tsai-

Table 1

Dependence of optimal designs on the chosen failure criteria for the loading $N_{xx} = 100$, $N_{yy} = N_{xy} = 0$ MPa m, and for two distinct fiber angles ($0 \leq j \leq 51$)

Failure criteria used to check feasibility	Optimal lay-up	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
Only Tsai–Wu criterion	$[-9_{25}/10_{22}]_s$	47	1.0007	0.9142
Only max stress criterion	$[5_{51}/\theta_0]_s$	51	0.6688	1.0168
Both Tsai–Wu and max. stress	$[0_j/0_{51-j}]_s$	51	1.0091	1.0091

Wu criterion is used, the optimal design is almost balanced and the angle imparting the highest strength is predicted around $\pm 10^\circ$, following the trend shown in Fig. 3. According to the maximum stress criterion, however, this configuration is unsafe. If only the maximum stress criterion is used, the optimal laminate is unidirectional with a fiber orientation angle of 5° , conforming to the trend shown in Fig. 2. According to the Tsai–Wu criterion, this design is highly nonconservative. If the Tsai–Wu and maximum stress criteria are used together, the optimal lay-up design conforms to the empirical observations; i.e. a laminate under uniaxial loading is strongest if all the fibers are aligned along the load direction.

For some other load cases, the optimal lay-ups having minimum thickness were obtained using the Tsai–Wu and maximum stress criteria together. A range of values were tried for the number of distinct fiber orientations. Table 2 shows the optimum angles, the number of plies oriented along these angles, and the total number of plies for a biaxially loaded laminate ($N_{xx} = 10$, $N_{yy} = 10$, $N_{xy} = 0$ MPa m) made of T300/5308. For this loading case, quite a number of multiple globally optimum lay-up designs were found. For two distinct fiber angles, the optimization algorithm found $[90_{47}/0_{47}]_s$, $[89_{47}/1_{47}]_s$, $[88_{47}/2_{47}]_s, \dots, [45_{47}/-45_{47}]_s, \dots, [1_{47}/-89_{47}]_s$ as the optimal lay-ups having the same objective function value. This means that the strength of a laminate having $[90_{47}/0_{47}]_s$ lay-up is the same for all in-plane biaxial loads having equal magnitude applied along any arbitrary x - y directions. For this loading case, tensile stresses transverse to the fibers are critical. Because for all these lay-ups, $[0_{47}/\theta - 90_{47}]_s$, transverse tensile stress in each ply is the same, they have the same safety factor, and thus the same objective function value. Among the globally optimal designs, $[90_{47}/0_{47}]_s$ and $[45_{47}/-45_{47}]_s$ are balanced lay-up sequences; hence they do not have shear-extension coupling. For T300/5208, optimal fiber orientations were found to be the same, $[\theta_j/\theta - 90_j]_s$; but the total number of plies is 14 as opposed to 94, because its tensile strengths are larger. Stacking sequence does not affect the strength of a symmetric laminate subject to in-plane loads since none of the equations, Eqs. (3), (5)–(8), depend on z coordinates of the plies. Therefore, $[40_{47}/-50_{47}]_s$ and $[50_{47}/-40_{47}]_s$, for example, have the same strength. In view of that, alternative stacking sequences are excluded from the results given in the tables.

Increasing the number of distinct fiber angles did not yield better lay-up designs. All of them have the same thickness (94 plies) and the same safety factor. However, larger numbers of distinct angles offered alternative designs. Among the optimal designs obtained using four distinct fiber angles, some of them are the same like $[65_{47}/39_0/13_0/-25_{47}]_s$, $[45_{20}/45_{27}/-45_{32}/-45_{47}]_s$, but some are different like $[90_{30}/0_{30}/45_{27}/-45_{27}]_s$. Although the optimal designs do not show any difference according to the fitness criterion adopted in this study, some of the designs may have more resistance to other forms of failure like buckling, fatigue, resonance, or better thermal properties, which may become critical for some applications. For this reason, being able to obtain all or most of the alternative optimal designs is important for a more comprehensive design process.

Table 2

The optimum lay-ups for the loading $N_{xx} = 10$, $N_{yy} = 10$, $N_{xy} = 0$ MPa m, and for various numbers of distinct fiber angles

Number of distinct fiber angles	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
2	$[\theta_{47}/\theta - 90_{47}]_s$; $[90_{47}/0_{47}]_s$, $[60_{47}/-30_{47}]_s, \dots$	94	1.0009	1.0050
4	$[0_{47}/\alpha_0/\beta_0/\theta - 90_{47}]_s$; $[45_{47}/83_0/32_0/-45_{47}]_s, \dots, [\theta_n/\theta_{47-n}/\theta - 90_k/\theta - 90_{47-k}]_s$; $[20_{40}/20_7/-70_{20}/-70_{27}]_s$ $[\theta_n/\theta_{47-n}/\alpha_0/\theta - 90_{47}]_s$; $[20_{40}/20_7/42_0/-70_{47}]_s, \dots, [\theta_n/\theta - 90_n/\alpha_{47-n}/\alpha - 90_{47-n}]_s$; $[60_{20}/-30_{20}/90_{27}/0_{27}]_s$	94	1.0009	1.0050
8	$[-11_{26}/79_{26}/-12_{20}/78_{20}/-17_1/73_1/\alpha_0/\beta_0]_s$	94	1.0009	1.0050
16	$[-38_1/52_1/-35_{28}/55_{28}/-36_6/54_6/-37_{12}/53_{12}/\alpha_0/\beta_0/\theta_0/\lambda_0/\varphi_0/\delta_0/\gamma_0]_s$	94	1.0009	1.0050

The material is T300/5308. ($\alpha, \theta, \beta, \dots$ are arbitrary angles between -90° and 90° , k and n are arbitrary positive integers less than 48).

The algorithm can find a global design or a near-global design in every run even with a large number of optimization variables. All the optimal designs had the same thickness of 94 plies with a safety factor of at least 1.00089 for Tsai–Wu, 1.0049 for the maximum stress criterion. This shows the reliability of the algorithm in finding the best solution among countless local optimums. Moreover, because the DSA algorithm uses N number of current configurations, it may find many multiple globally optimum designs in a single run.

Table 3 shows the results for the case of pure shear loading for two distinct fiber angles. Using a larger number of distinct fiber angles did not result in a different lay-up sequence. For different materials, different optimal lay-up sequences were obtained. For these loading cases, because the safety factor for Tsai–Wu, SF_{TW} , was much smaller than that of the maximum stress criterion, SF_{MS} , the weight for the Tsai–Wu criterion (w_2) was increased. Otherwise, the algorithm might choose laminate designs with a larger SF_{MS} , but a lower SF_{TW} .

Table 4 shows the results for the case of pure shear loading; but this time the magnitudes of the compressive strengths are taken the same as the tensile strengths, ($X_c = -X_t$, $Y_c = -Y_t$). This is the way some researchers adopt to avoid false optimums when the Tsai–Wu criterion is used. However, this approach may yield extremely conservative designs as seen in the table. The optimal design for T300/5308 is $[0_n/90_{210-n}]_s$. Because shear stresses are critical and the magnitudes of the shear stresses in 0° and 90° plies are the same, any combination of 0° and 90° plies with a total number of 210 plies is optimal. This thickness is far larger than the thickness of the real optimum laminate, which is 81 plies as given in

Table 3

The optimum lay-ups obtained using two distinct fiber angles for the loading $N_{xx} = 0$, $N_{yy} = 0$, $N_{xy} = 40$ MPa m.

Material	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
T300/5208	$[-45_{14}/45_{27}]_s$	41	1.0150	1.4577
T300/5308	$[-63_{27}/48_{54}]_s$, $[-27_{27}/42_{54}]_s$	81	1.0032	1.1637

Table 4

The optimum lay-ups obtained using only tensile strengths ($X_c = -X_t$, $Y_c = -Y_t$) for the loading $N_{xx} = 0$, $N_{yy} = 0$, $N_{xy} = 40$ MPa m

Material	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
T300/5208	$[-45_{23}/45_{23}]_s$	43	1.0206	1.4615
T300/5308	$[0_n/90_{210-n}]_s$; $[0_{105}/90_{105}]_s$, $[0_{10}/90_{200}]_s$	210	1.0001	1.0001

Table 3. The optimal design for T300/5208, on the other hand, is $[-45_{23}/45_{23}]_s$. Because, the compressive and tensile strengths along the fiber direction are the same and the tensile stresses transverse to the fibers are critical, taking $Y_c = -Y_t$ did not much affect the optimal configuration. Since the Tsai–Hill criterion does not use compressive strengths of laminates, it also yield the same nonconservative results.

Table 5 shows the optimal laminate designs for various biaxial loading cases obtained using two distinct fiber angles. For the loading case $N_{xx} = 10$, $N_{yy} = 5$, $N_{xy} = 0$ MPa m, the optimal lay-up is $[37_{27}/-37_{27}]_s$ with 54-ply thickness. When N_{xx} is increased to 20 MPa m, strangely the thickness of the optimal laminate becomes smaller. This counter intuitive result can be explained by considering the differences in the stress states. When N_{xx} is increased to 20 MPa m and the laminate design is changed to $[31_{23}/-31_{23}]_s$, ϵ_{xx} increases from 0.323×10^{-2} to 0.729×10^{-2} , ϵ_{yy} , on the other hand, turns from tension to (8.39×10^{-5}) compression (-0.237×10^{-2}) due to Poisson’s effect. The stress transverse to the fibers then decreases from 18.49 MPa to 15.85 MPa, while the other principal stresses (shear stress and normal stress along the fiber direction) increase. Because, the transverse tensile stresses are critical, a thinner laminate could carry a larger load. When N_{xx} is increased to 40 MPa m, the same trend continues. However, when it is increased to 80 or 120 MPa m, a thicker laminate is required.

Table 6 shows the optimal laminate designs for load cases in which shear load is increased. One interesting loading case is $N_{xx} = 10$, $N_{yy} = 10$, $N_{xy} = 10$ MPa m. In comparison to the thickness of the optimal laminate for the loading case $N_{xx} = 10$, $N_{yy} = 10$, $N_{xy} = 0$ MPa m, which is 94 plies as given in Table 2, the thickness is quite low (11 plies). The reason is that the application of N_{xy} , causes the principal shear and transverse stresses disappear for a fiber oriented with an angle of 45° . Only tensile stresses along the fiber direction remain, for which the plies are strongest. When the shear loading is increased, the thickness increases, because this condition is disrupted. As seen in the table, for three distinct angles a better design is obtained, which have the same thickness but a larger safety factor.

Table 5

The optimum lay-ups obtained using two distinct fiber angles for various biaxial loading cases

Loading: $N_{xx}/N_{yy}/N_{xy}$ (MPa m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
10/5/0	$[37_{27}/-37_{27}]_s$	54	1.0068	1.0277
20/5/0	$[31_{23}/-31_{23}]_s$	46	1.0208	1.1985
40/5/0	$[26_{20}/-26_{20}]_s$	40	1.0190	1.5381
80/5/0	$[21_{25}/-19_{28}]_s$	53	1.0113	1.2213
120/5/0	$[17_{35}/-17_{35}]_s$	70	1.0030	1.0950

The material is T300/5308.

Table 6
The effect of increasing shear load

Loading: $N_{xx}/N_{yy}/N_{xy}$ (MPa m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
10/10/10	[45 ₁₁ /α ₀] _s	11	1.0883	1.0883
10/10/20	[45 ₃₀ /–12 ₄] _s	34	1.0127	1.1489
	[45 ₃₀ /–78 ₄] _s			
	[44 ₂₇ /43 ₃ /–76 ₄] _s , [46 ₂₇ /47 ₃ /–14 ₄] _s	34	1.0132	1.1256
10/10/40	[45 ₆₀ /–22 ₁₈] _s	78	1.0103	1.2076
	[45 ₆₀ /–68 ₁₈] _s			
	[45 ₃ /44 ₅₆ /–21 ₁₉] _s , [45 ₃ /46 ₅₆ /–69 ₁₉] _s	78	1.0121	1.2093
10/10/80	[43 ₁₁₃ /–24 ₄₇] _s	160	1.0053	1.1853
	[47 ₁₁₃ /–66 ₄₇] _s			
	[43 ₁₁₁ /44 ₂ /–24 ₄₇] _s , [47 ₁₁₁ /46 ₂ /–66 ₄₇] _s	160	1.0063	1.1850

The material is T300/5308.

Table 7
The effect of increasing compressive load

Loading: $N_{xx}/N_{yy}/N_{xy}$ (MPa m)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
10/–10/0	[3 ₁₄ /72 ₇] _s	21	1.0404	1.2068
10/–20/0	[3 ₁₈ /82 ₁₂] _s	30	1.0156	1.1253
	[4 ₁ /5 ₁₅ /12 ₁ /80 ₁₃] _s	30	1.0172	1.1352
10/–40/0	[1 ₂₀ /89 ₂₅] _s	45	1.0219	1.1104
	[0 ₃ /–1 ₁₇ /87 ₂ /–89 ₂₃] _s	45	1.0226	1.1112
10/–80/0	[3 ₁₈ /90 ₅₄] _s	72	1.0150	1.1026
	[–4 ₁₈ /–87 ₂ /89 ₁ /90 ₅₀] _s	71	1.0007	1.0871
10/–10/10	[19 ₁₈ /–48 ₁₁] _s , [26 ₁₈ /–87 ₁₁] _s	29	1.0096	1.1855
	[21 ₁₅ /22 ₄ /–50 ₁₀] _s	29	1.0145	1.1888
	[23 ₄ /24 ₁₅ /–85 ₁₀] _s			
10/–20/10	[20 ₂₁ /–84 ₁₅] _s	36	1.0056	1.1331
	[20 ₄ /21 ₁₆ /–85 ₁₆] _s	36	1.0058	1.1410
10/–40/10	[9 ₂₃ /–76 ₂₆] _s	49	1.0095	1.1020
10/–80/10	[5 ₂₀ /–84 ₅₄] _s	74	1.0050	1.0915
	[4 ₂₀ /–83 ₂₅ /–84 ₂₉] _s	74	1.0056	1.0924

The material is T300/5308.

Table 8
The optimum lay-ups for the loading $N_{xx} = 40$, $N_{yy} = 5$, $N_{xy} = 0$ MPa m, and for various numbers of distinct fiber angles

Interval between orientation angles (°)	Optimum lay-up sequences	Half laminate thickness	Safety factor for Tsai–Wu	Safety factor for max. stress
1	[26 ₂₀ /–26 ₂₀] _s	40	1.0190	1.5380
5	[25 ₂₀ /–25 ₂₀] _s	40	1.0112	1.6914
10	[30 ₁₇ /–20 ₂₆] _s	43	1.0100	1.8556
15	[30 ₂₃ /–30 ₂₃] _s	46	1.0397	1.2318
30	[30 ₂₃ /–30 ₂₃] _s	46	1.0397	1.2318
45	[0 ₁₀₇] _s	107	1.0093	1.0328

The material is T300/5308.

Table 7 shows the optimal laminate designs for various load cases in which compressive load is increased. With increasing load, the required minimum thickness also increases. For the loading case $N_{xx} = 10$, $N_{yy} = -80$, $N_{xy} = 0$ MPa m, a better design with a smaller thickness was obtained with four distinct lamina angles in comparison to the design with two distinct lamina angles.

Table 8 shows the optimal designs obtained using various intervals between the fiber orientation angles for the loading $N_{xx} = 40$,

$N_{yy} = 5$, $N_{xy} = 0$ MPa m. As the results indicate, with a smaller interval, one may obtain better designs. Choosing large intervals may lead to gravely inferior designs. If only 0°, ±45°, and 90° angles are allowed, almost three times thick laminates are required to bear the applied load.

4.1. Comparison with the results of a nonlinear programming

As far as the authors know, there is no study that formulated the problem as in the present study. An investigation conducted by Wang and Karihaloo [7] is similar to this study in that they considered composite laminates subject to in-plane static loads and they used lamina thickness and orientation angles as design variables. On the other hand, they aimed at maximizing the strength of a laminate rather than minimizing its total thickness. Because they used a deterministic local search algorithm that employed a nonlinear programming, comparison of their results with the results obtained using the algorithm proposed in the present study may indicate the advantage gained by global search algorithms. The present algorithm was modified to maximize the safety factor and optimal lay-ups were obtained for a number of load cases. Tables 9 and 10 show optimal lay-up sequences for four and eight distinct lamina angles, respectively. Lamina thickness is not varied and equal to four as in some of the example problems that Wang and Karihaloo [7] considered. Although many multiple globally optimum lay-ups were obtained, only one lay-up was given for each load case. As seen in the tables, the global search algorithm managed to find better lay-up sequences. The algorithm found [45_n] as the optimal laminate design for the loading case of $N_{xx} = N_{yy} = N_{xy}$. This design has a very large safety factor. This is because Wang and Karihaloo [7] used a failure criterion based on fracture mechanics taking into account transverse cracks that are susceptible to cause failure under shear stress and transverse tensile normal stress only. Accordingly, a laminate with [45_n] lay-up sequence subject to $N_{xx} = N_{yy} = N_{xy}$ has theoretically infinite strength since only normal stresses along the fiber direction exist. Therefore, the result of the optimization for this loading case is not surprising. Wang and Karihaloo [7] considered lamina thickness as a continuous optimization variable in some other design optimization problems. Because the present algorithm was developed considering thickness as a discrete variable, it was not possible to compare the results of the optimization problems in which fiber angles together with thickness were varied.

4.2. Conclusions

In this study, an optimization methodology for weight minimization of composite plates under in-plane loading was presented. Methods were proposed in order to overcome the difficulties faced by the previous researchers. The direct simulated annealing algorithm (DSA) was adopted as search algorithm. For some loading cases, many global or near-global optimum designs were found to exist. The algorithm proved to be quite reliable in locating these designs. In a single optimization run, the algorithm could find one or more of them even with a large number of design variables.

When the Tsai–Wu or maximum stress failure criteria is used individually, an optimization algorithm is lead to false optimal designs because of the particular features of their failure envelopes. On the other hand, when they are used together, false optimums are avoided. Use of the Tsai–Hill criterion, which includes only tensile strengths, or taking compressive strengths equal to the tensile strengths leads to overly conservative designs.

For different materials, fiber orientation angles in the optimal lay-up designs may be different for the same loading case. Thus, one may not generalize the results obtained for a particular material to others.

Table 9
Comparison with the results of a nonlinear programming [7]

Loading: $N_{xx}/N_{yy}/N_{xy}$ (kPa m)	Optimum lamina orientations		Safety factor	
	Wang and Karihaloo [7]	Present study	Wang and Karihaloo [7]	Present study
200/200/0	[62.68 ₄ /-54.20 ₄ /81.23 ₄ /-1.96 ₄] _s	[50.80 ₄ /-49.80 ₄ /26.59 ₄ /-49.73 ₄] _s	1.59	2.14
200/0/200	[32.30 ₄ /-56.61 ₄ /-7.78 ₄ /33.87 ₄]	[31.72 ₁₆]	1.86	4.84
400/200/0	[-27.46 ₄ /57.58 ₄ /-43.61 ₄ /20.11 ₄]	[30.98 ₄ /-36.57 ₄ /37.67 ₄ /-37.20 ₄]	1.34	1.64
200/200/200	[45.45 ₄ /51.72 ₄ /43.38 ₄ /39.58 ₄]	[45.00 ₁₆]	10.21	1.11×10^{16}

Lamina thickness is constant and equal to four plies. No of distinct fiber angles is four.

Table 10
Comparison with the results of a nonlinear programming [7]

Loading: $N_{xx}/N_{yy}/N_{xy}$ (kPa m)	Optimum lamina orientations		Safety factor	
	Wang and Karihaloo [7]	Present study	Wang and Karihaloo [7]	Present study
400/400/0	[-3.08 ₄ /-90.00 ₈ /-28.51 ₄ /88.00 ₄ /-28.49 ₄ /-27.59 ₄ /40.05 ₄] _s	[-73.68 ₄ /33.57 ₄ /-50.69 ₄ /30.89 ₄ /-51.55 ₈ /30.75 ₈] _s	1.70	2.15
400/0/400	[36.66 ₄ /-57.68 ₄ /36.47 ₄ /90.00 ₄ /37.31 ₄ /24.58 ₄ /-57.77 ₄ /31.65 ₄] _s	[31.72 ₁₆]	2.22	4.84
800/400/0	[-26.94 ₄ /49.26 ₄ /-37.54 ₄ /40.09 ₄ /-42.78 ₄ /55.36 ₄ /-24.16 ₄ /10.86 ₄] _s	[-29.94 ₄ /35.60 ₄ /-36.69 ₄ /36.31 ₁₂ /-36.75 ₈] _s	1.40	1.65
400/400/400	[49.29 ₈ /49.27 ₄ /26.60 ₄ /49.42 ₄ /11.91 ₄ /50.14 ₄ /49.33 ₄] _s	[45.00 ₁₆]	3.62	3.86×10^{15}

Lamina thickness is constant and equal to four plies. No of distinct fiber angles is eight.

In some cases, the optimal designs can be counter intuitive. Sometimes, when one component of loading is increased, load bearing capacity of the optimized laminate may increase. Therefore, a design process for composite materials should not be based on intuition or experience.

Usually, choosing only two distinct fiber angles is sufficient to obtain the best possible design. In some load cases, different lay-up sequences with the same objective function value were obtained with a large number of distinct fiber angles. In some others, however, better designs were obtained with three or four distinct angles, which have the same thickness but a larger safety factor. In one case, a thinner laminate was obtained with four distinct angles.

If the available fiber orientations are scarce, extremely inferior designs may be obtained. This is the case, if only 0°, ±45°, and 90° angles are allowed. For this reason, the interval between the available angles should be small enough to be able to find the best design.

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