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Design optimization of laminated composites using a new variant of simulated annealing

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ABSTRACT

The aim of this study is to minimize the thickness (or weight) of laminated composite plates subject to both in-plane and out-of-plane loading. A new variant of the simulated annealing algorithm is proposed to optimize the lay-up design. Fiber orientation angle and number of plies in each lamina are used as design variables. Considering static failure as the critical failure mode, the maximum stress and Tsai–Wu criteria are used together to predict failure. Numerical results show that the optimization methodology proposed in this study can find the globally optimum laminate designs even with a high number of design variables.

1. Introduction

Use of composite materials has been increasing steadily in aerospace, automotive and other engineering applications due to their high specific stiffness and strength. However, because of their high cost, the composite structures should be optimized to make the best use of material. In this way, composite materials may become feasible alternatives as a replacement for conventional homogeneous materials in many applications. For this purpose, many researchers conducted studies to minimize the material use in composite structures by optimizing fiber orientations, lamina thickness, stacking sequence, or geometric parameters.

While minimizing the weight or thickness of composite structures, the designers need to consider all the design parameters, loading conditions, failure modes and computational assumptions. In typical engineering applications, composite structures are under various types of loading conditions, not only in-plane loads but also out-of-plane loads such as auto-body chassis or airplane fuselage and wings. In this respect, a model accounting for only in-plane loads fails to capture the physics of the phenomena; as a result it is essential to consider all the loading types in the analysis and design optimization of composite plates. However, in most of the studies on the optimization of laminated composite plates only in-plane loads were considered. The studies accounting for out-of-plane loads [1–36], either being bending and/or twisting moments [1–6,13,16,19,20,24,25,27,32,35,36] or transverse loads [7–12,14,15,17,18,20–23,26,28–31,33,34], were relatively rare. In some of these studies the objective was to minimize thickness [2,12,15,17–19,21,22,28], weight [3,6–8,11,20,35], cost and weight [29,30,36], thickness and change in the strain energy [4], or maximize the static strength of composite laminates for a given thickness [5,14,16,18,25,26,30,34], strength-to-weight ratio [5], stiffness [1,5,7,9–11,13,17,22,27,31], energy absorption capacity [33] twisting angle at plate tip to reduce aerodynamic loading [24], or maximizing static strength while minimizing weight [32]. In the present study, laminate thickness was minimized; but by modifying the objective function, the same optimization procedure can be applied to design optimization problems in which different criteria are used for the effectiveness of the laminate design. Although plain laminates are considered in this study, the algorithm can be applied to hybrid laminates or sandwich plates by introducing minor changes to the optimization algorithm. Because, coupling between bending and extension is not desired in most of the applications of composite materials, only symmetric laminates are considered in this study. In rare applications like fan or helicopter blades, favorable use of the coupling can be made [37]. The extension of the structural analysis method for such applications is straightforward.

In order to evaluate the objective and constraint functions and thus estimate the structural performance of the candidate optimal designs generated during the optimization process, structural analysis of the laminate should be carried out. The most frequently adopted approach is to use either the classical lamination theory (CLT) [1–6,16,24,25,27,29,31,32,35,36], or finite element method (FEM) [8–12,14,15,17,19–22,26,28,30,33,34]. For laminates having relatively simple geometries such as smooth rectangular or circular plates without any discontinuities like holes and having small

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thickness relative to lateral dimensions, CLT is often preferred. As for complex geometries such as stiffened plates [20,33] and plates with a hole [38], FEM is more suitable. Considering that composites optimization problems have complex solution domains with high numbers of optimization variables, many iterations are required to locate the globally optimum design. Because the whole structure should be analyzed with FEM, very long run-times and highly upgraded computer architectures are required to conduct composite optimization studies. One may use general structural analysis software to determine the loading state at critical locations, then using these as input loading, CLT can be applied for the analysis of different lay-up configurations generated during the optimization process. Because the objective of the present study is to propose an optimization methodology to find globally optimal laminate designs through a stochastic search, the computationally efficient CLT is employed for structural analysis.

In composite laminate design, lamina angle and thickness are the main design variables. In some of the previous studies of design optimization, lamina thickness [1–5,7–12,17–22,27,29,35] and/or fiber orientation angles [1,4,6,8–12,14,17,18,21,26,27,29] were taken to be continuous design variables. Considering that composite laminates are fabricated from prepregs with a given thickness, a discrete value should be specified for the lamina thickness. Rounding the optimal values to the nearest discrete manufacturable value may lead to less than optimal designs or even to constraint violations. Besides, manufacturing precision dictates the possible fiber orientations. Fiber orientations are chosen from a finite set of angles during the design process because of the difficulty of exactly orienting fibers along a given direction. For these reasons, a discrete optimization technique is used in the present study so that angle and thickness of laminae take discrete values.

While minimizing the weight of a composite laminate, its feasibility should be checked by imposing a strength constraint in order to ensure that the resulting optimally designed laminate will carry the applied loads without failure. The previous researchers adopted the first-ply-failure approach using the Tsai–Wu [2,6,8,15,17,19,20,24–28,32–34,36], Tsai-Hill [3,12,14,21,30], the maximum stress [33,36], or the maximum strain [4] static failure criteria. In the present study, the suitability of using the Tsai–Wu and the maximum stress criteria in composites optimization was investigated and use of both of them in the optimization process was attempted to avoid false optimums.

The main difficulty in composite optimization problems is the existence of immense number of locally optimum designs. With a high number of design variables, local optimums dramatically increase in number. Because a locally optimum design may significantly be worse than the globally optimal design, an effective optimization procedure should aim to find the global optimum. In some of the previous studies, global search algorithms were used like genetic algorithms [5,24,26,28,30,32–35], ant colony optimization [31], and branch and bound [16]. On the other hand, simulated annealing (SA), which is known to be one of the most reliable search algorithms in locating the globally optimal point, found few applications in composites optimization [39–43]. If the design variables are few or just an improvement over the current design is desired, deterministic local search algorithms [2–4,7–12,14,17–22,25,27,29], which may be coupled with a multi-start optimization approach [15], analytical methods [13], or parametric studies [33] may be viable approaches. However, if the number of design variables is large, it is very unlikely that the resulting design will be the globally optimal design, even though considerable improvement can be achieved over the current design with a local search algorithm.

This study is an extension of a previous study [43] conducted by the authors. One of the main differences lies in that in the previous study only in-plane loading was considered, while in the present study both in-plane and out-of-plane loading is considered. If a laminate is subjected to an out-of-plane load, not only existence of a lamina with a certain thickness and orientation affects the mechanical response of the laminate but also its position within the laminate. For this reason, in design optimization not only the set of lamina thicknesses and fiber angles is optimized but also the stacking sequence. This will dramatically increase the number of distinct configurations and thus the difficulty of global search. This requires a more reliable search algorithm than the one used in the previous study. Accordingly, in the present study, a new variant of simulated annealing (SA) algorithm is proposed to search the global optimum. The basic features of SA, which are the use of a temperature parameter to control convergence and evaluation of acceptability based on Boltzmann distribution [44], are adopted. Besides, a population of current configurations is used instead of a single current configuration as in the direct search simulated annealing (DSA) [45]. On the other hand, a number of modifications are introduced in the generation mechanism of new configurations, replacement scheme of accepted configurations, and the reduction scheme (or cooling schedule) for the temperature parameter. Through these changes, first of all, the reliability and efficiency of SA algorithm are increased. Secondly, convergence is made dependent on a single parameter. SA like the other stochastic global search algorithms requires many trials in order to thoroughly search the feasible domain. Effectiveness of the search process depends on the values of the parameters defining the cooling schedule, replacement scheme, and generation mechanism. Finding appropriate values for the parameters of the problem at hand is time consuming. By making all the parameters dependent on a single parameter, adaptation of the algorithm to other problem areas is made easier. After introducing these changes, global optimization of composite laminates with a much larger number of distinct laminae, i.e. with a much larger design domain, in comparison to the previous studies can be reliably achieved.

2. Problem formulation

2.1. Problem statement

Consider a structure made of orthotropic layers perfectly bonded together and reinforced by continuous fibers. The structure is symmetric with respect to its mid-plane. This multilayered structure is subjected to in-plane normal (N_{xx} and N_{yy}) and shear (N_{xy} and N_{yx}) loading as well as bending (M_{xx} and M_{yy}) and twisting moment resultants (M_{xy} and M_{yx}) as shown in Fig. 1.

The objective is to minimize the laminate thickness, t, with the condition that it does not fail under the applied static loads. The problem can be stated in general terms as

$$\begin{aligned} &\text{Minimize} \quad t = \text{thickness} \\ &\text{Subject to failure criteria} \end{aligned} \tag{1}$$

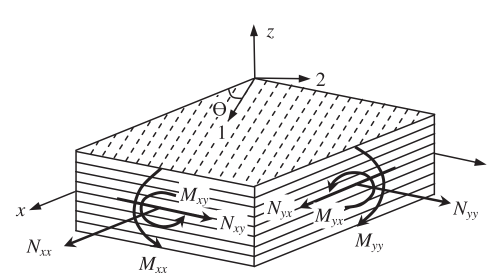

Fig. 1. A schematic of the composite structure and the loading considered in this study.

The thickness of each ply in the laminate is the same and not varied during the optimization, because laminates are usually made of prepregs with a given thickness. The number of distinct fiber orientation angles, m, is specified by the designer, while the orientation angles, θ_k, and how many plies, n_k, are oriented with angle θ_k are taken as design variables; their values are to be determined during the optimization process. A stack of contiguous plies oriented in the same direction is called lamina. θ_1 is the orientation angle of the outermost lamina and θ_m is the innermost lamina below the mid-plane as shown in Fig. 2. The laminate thickness can be expressed as

$$t = 2t_o \sum_{k=1}^{m} n_k \tag{2}$$

where t_o is the thickness of an individual ply, n_k is the number of plies with a fiber orientation of θ_k. The factor '2' appears because of the symmetry with respect to the middle plane. Considering that the plies are made of the same material, minimizing the thickness leads to the same optimum configuration as the minimization of weight.

The orientation angles take discrete values; they are chosen from a given set of angles. According to the manufacturing precision, the interval between the consecutive angles may be 15°, 10°, 5°, 1°, or even smaller.

2.2. Analysis of composite laminates subject to in-plane and out-of-plane loads

If the thickness is small relative to the lateral dimensions and deformations are small, the mechanical behavior of a composite laminate can be correctly analyzed using the classical lamination theory. Each ply is assumed to be under plane stress condition and, therefore, out-of-plane stress components are taken as zero ($\sigma_{zz} = \tau_{xz} = \tau_{yz} = 0$). In-plane loads induce a uniform strain distribution through the thickness, ε_{ij}^0, while out-of-plane loads induce a strain state varying linearly through the thickness and depending on the curvature. Superimposing them, one may express the strain state as

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{3}$$

where ε_{xx}^0, ε_{yy}^0, and γ_{xy}^0 are the mid-plane strains, κ_{xx}, κ_{yy}, and κ_{xy} are the curvature terms. The stress components are related to the strain components as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}_k \tag{4}$$

where k is the lamina number counted from the bottom, $\overline{Q}_{ij}$ are the off-axis stiffness components, which depend on the fiber orientation, θ_k, and elastic properties of the material along the principal directions, E_1, E_2, G_{12}, ν_{12}, and ν_{21} [46]. The stress components in the kth lamina are obtained by substituting Eq. (3) into Eq. (4).

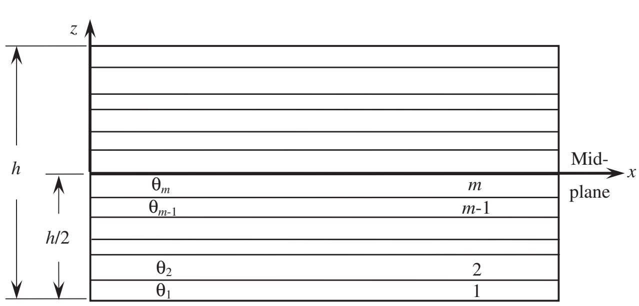

Fig. 2. A schematic of a symmetric laminate configuration.

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_{x} \\ \kappa_{y} \\ \kappa_{xy} \end{Bmatrix} \right) \tag{5}$$

Because the stress components, σ_{ij}, depend on the z coordinate, not only lamina thickness and lamina angles but also stacking sequence of the laminae affects the mechanical response of the laminate in the case of out-of-plane loading. Accordingly, not only the existence of a lamina with a certain fiber angle but also its location is important during optimization.

Stress resultants (forces and bending moments per unit lateral length of a cross section) are obtained by through-the-thickness integration of the stresses in each ply.

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} dz = \sum_{k=1}^{2m} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k dz \tag{6}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} z dz = \sum_{k=1}^{2m} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix}_k z dz \tag{7}$$

Here m is the number of distinct laminae in one of the symmetric portions above or below the mid-plane. Substituting the stress–strain relation given by Eq. (5) into Eqs. (6) and (7), one obtains

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{yy}^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix} \tag{9}$$

where A_{ij} are membrane stiffness components, D_{ij} are bending stiffness components, and B_{ij} are bending-extension coupling stiffness components given by

$$A_{ij} = \sum_{k=1}^{2m} (\overline{Q}_{ij})_k (z_k - z_{k-1}) \tag{10}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{2m} (\overline{Q}_{ij})_k (z_k^2 - z_{k-1}^2) \tag{11}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{2m} (\overline{Q}_{ij})_k (z_k^3 - z_{k-1}^3) \tag{12}$$

As seen in Eqs. (8) and (9), the response of the laminate to membrane and bending forces are coupled for nonzero B_{ij}. If symmetric laminates are considered as in this study, B_{ij} terms become zero as Eq. (11) implies.

Given the in-plane loading, N_{xx}, N_{yy}, and N_{xy}, and out-of-plane loading, M_{xx}, M_{yy}, and M_{xy}, one may obtain the mid-plane strains ε_{xx}^0, ε_{yy}^0, and γ_{xy}^0, and the curvature terms, κ_{xx}, κ_{yy}, are κ_{xy}, using Eqs. (8) and (9). Then, the off-axis stress components in each ply σ_{xx}^k, σ_{yy}^k, and τ_{xy}^k can be calculated using Eq. (5). After that, the principal stress components, σ_{11}^k, σ_{22}^k, and τ_{12}^k, can be calculated using the transformation rules [46].

Based on the principal stress components, σ_{11}^k, σ_{22}^k, and τ_{12}^k, one may judge whether the *k*th ply will fail or not using appropriate static failure criteria.

2.3. Static failure criteria

During weight minimization of a composite structure, strength constraints should be imposed, because decreasing the number of load carrying plies eventually leads to failure. Whenever a new lay-up design is generated by the search algorithm during optimization, its feasibility regarding load-carrying capacity should be checked. In this study, only the static failure modes are assumed to be critical for the laminate. The other failure modes, fatigue, low stiffness, buckling, delamination, resonance etc. are assumed to be not critical.

One of the commonly used approaches to estimate the load carrying capability of a laminate design is to use a limit theory such as the maximum stress criterion. According to this criterion, failure is predicted whenever one of the principal stress components exceeds its respective strength. The failure envelope for a ply under in-plane normal and shear stresses is then defined by the following inequalities:

$$\sigma_{11} < X_t \quad \text{and} \quad \sigma_{11} > X_c \quad \text{and} \quad \sigma_{22} < Y_t \quad \text{and} \quad \sigma_{22} > Y_c \quad \text{and} \quad |\tau_{12}| < S \tag{13}$$

where "*X*" and "*Y*" symbolize the strengths along the fiber direction and transverse to it, respectively; the subscripts "*t*" and "*c*" denote the tensile and compressive strengths; *S* is the ultimate in-plane shear strength of the laminate under pure shear loading. Adopting the first – ply – failure criterion, the whole laminate is considered to have failed, if one of these inequalities is not satisfied for any one of the laminae.

The safety factor of a structure is an indication of its load carrying capacity. Values less than 1.0 indicate failure. In order to calculate the safety factor of a laminate based on the maximum stress criterion, first, the principal stresses (σ_{11}^k, σ_{22}^k, and τ_{12}^k) in each lamina are determined; the safety factor for each failure mode is calculated; then the minimum of them is denoted as the safety factor of the lamina, SF_{MS}^k.

$$SF_{MS}^k = \min \text{ of} \begin{cases} SF_X^k = \begin{cases} X_t/\sigma_{11} & \text{if } \sigma_{11} > 0 \\ X_c/\sigma_{11} & \text{if } \sigma_{11} < 0 \end{cases} \\ SF_Y^k = \begin{cases} Y_t/\sigma_{22} & \text{if } \sigma_{22} > 0 \\ Y_c/\sigma_{22} & \text{if } \sigma_{22} < 0 \end{cases} \\ SF_S^k = S/|\tau_{12}| \end{cases} \tag{14}$$

Then, the minimum of the lamina safety factors, SF_{MS}^k, is denoted as the safety factor of the laminate SF_{MS}.

$$SF_{MS} = \min \text{ of } SF_{MS}^k \text{ for } k = 1, 2, \ldots, m-1, m \tag{15}$$

Fig. 3 shows the safety factors calculated using this criterion for a laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for a range of fiber orientation angles, θ. Two different laminate lay-up configurations are considered. One is a balanced and symmetric laminate, $[\theta_{25}/-\theta_{25}]_s$, the other is a unidirectional laminate, $[\theta_{50}]_s$. The graph indicates the change in the safety factor with orientation angle θ. One may observe that for the balanced laminate, $[\theta_{25}/-\theta_{25}]_s$ under uniaxial loading (N_{xx} = 9.8 MPa m), the criterion correctly predicts that the laminate is strongest for $\theta = 0°$, in which fibers are oriented along the loading direction. The safety factor for this case, which is 1.01, is the highest of all. However, for the unidirectional laminate, $[\theta_{50}]_s$, the criterion falsely predicts the highest safety factor for $\theta = 5°$. This means that an optimization process in which failure is assessed based on the maximum stress criterion may stick to a spurious optimum design for an unbalanced laminate. On the other hand, as Fig. 4 shows, the highest safety factors are calculated at angles other than $\theta = 0°$ for a laminate subjected to bending moment M_{xx}. Therefore the maximum stress criterion by itself is not suitable for a general laminate design optimization procedure.

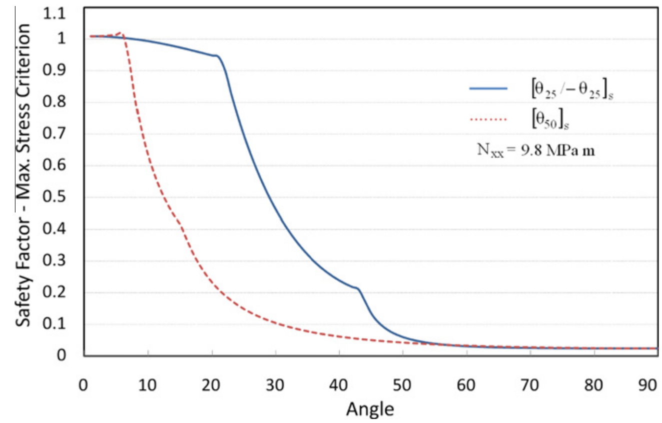

Fig. 3. Safety factors calculated using the maximum stress criterion for a laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for a range of fiber orientation angles, θ.

The general form of the Tsai–Wu failure criterion for orthotropic materials under 2D stress state is expressed as [46,47]

$$\frac{\sigma_{11}^2}{X_t|X_c|} + \frac{\sigma_{22}^2}{Y_t|Y_c|} + \frac{\tau_{12}^2}{S^2} - \frac{\sigma_{11}\sigma_{22}}{\sqrt{X_t X_c Y_t Y_c}} + \left(\frac{1}{X_t} - \frac{1}{|X_c|}\right)\sigma_{11} + \left(\frac{1}{Y_t} - \frac{1}{|Y_c|}\right)\sigma_{22} < 1 \tag{16}$$

The left hand side of the equality in Eq. (16) is called failure index. If it is less than 1.0, the structure will not fail; the closer it is to zero, the safer will be the laminate. In this study, safety factor is used as an indication of the strength of the laminate instead of failure index. The safety factor for the *k*th lamina, SF_{TW}^k, according to the Tsai–Wu criterion is defined as the multiplier of the stress components at lamina *k*, σ_{ij}^k, that makes the right hand side of Eq. (16) equal to 1.0. Eq. (16) then becomes

$$a(SF_{TW}^k)^2 + b(SF_{TW}^k) - 1 = 0 \tag{17}$$

where

$$a = \frac{(\sigma_{11}^k)^2}{X_t|X_c|} + \frac{(\sigma_{22}^k)^2}{Y_t|Y_c|} + \frac{(\tau_{12}^k)^2}{S^2} - \frac{(\sigma_{11}^k)(\sigma_{22}^k)}{\sqrt{X_t X_c Y_t Y_c}} \tag{18}$$

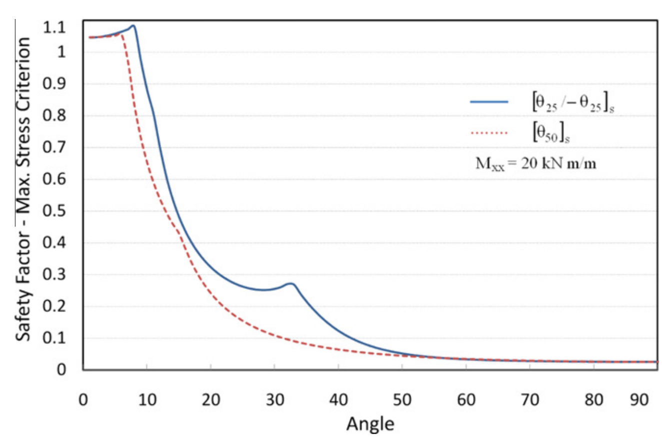

Fig. 4. Safety factors calculated using the maximum stress criterion for a laminate subjected to one component of bending moment (only $M_{xx} \neq 0$) for a range of fiber orientation angles, θ.

$$b = \left(\frac{1}{X_t} - \frac{1}{|X_c|}\right)\sigma_{11}^k + \left(\frac{1}{Y_t} - \frac{1}{|Y_c|}\right)\sigma_{22}^k \quad (19)$$

The root of Eq. (17) gives the safety factor. Because a negative safety factor is not physically meaningful, the absolute value of the first root is considered as the actual safety factor.

$$\mathrm{SF}_{TW}^k = \left|\frac{-b + \sqrt{b^2 + 4a}}{2a}\right| \quad (20)$$

Then, the minimum of SF_{TW}^k is chosen as the safety factor of the laminate

$$\mathrm{SF}_{TW} = \min \text{ of } \mathrm{SF}_{TW}^k \text{ for } k = 1, 2, \ldots, m-1, m \quad (21)$$

Fig. 5 shows the safety factor calculated using the Tsai–Wu criterion for two laminates subjected to uniaxial loading (only $N_{xx} \neq 0$). For the unidirectional laminate, $[\theta_{50}]_s$, the criterion correctly predicts that the laminate is strongest for $\theta = 0°$. The safety factor quickly decays with increasing θ. However, for the balanced laminate, $[\theta_{25}/-\theta_{25}]_s$, the criterion falsely estimates the highest safety factor as 1.087 at $\theta = 10°$. Actually, the laminate is expected to become weaker and weaker when θ is increased. One may conclude that the Tsai–Wu criterion may also lead to false optimum designs in an optimization process. Another known optimum design is $[45_n/-45_n]_s$ for the loading $N_{xx} = N_{yy} = N_{xy}$, which both criteria correctly predict. As opposed to the maximum stress criterion, the Tsai–Wu criterion correctly estimates that the laminate is strongest for $\theta = 0°$ if it is subjected to bending moment M_{xx} as shown in Fig. 6.

Considering the predictions of these two failure criteria for the two different laminate designs under two different loading conditions, each criterion seems to compensate the shortcomings of the other. If one of them incorrectly predicts the trend of strength for a given laminate configuration, the other correctly predicts. By enforcing the satisfaction of both criteria and calculating the safety factor using both criteria, one may correctly find the optimum designs. As Fig. 7 shows, a combined safety factor better conforms to the trend of the in-plane laminate strength, which continuously decreases when the angle between the fibers and the load axis gets larger. The combined safety factor is obtained by adding 90% and 10% of the safety factors calculated according to the maximum stress and the Tsai–Wu criteria, respectively. In this study, both the maximum stress and the Tsai–Wu criteria are used to assess the load bearing capacity of composite laminates with the expectation that false optimum designs will be avoided for any laminate configuration.

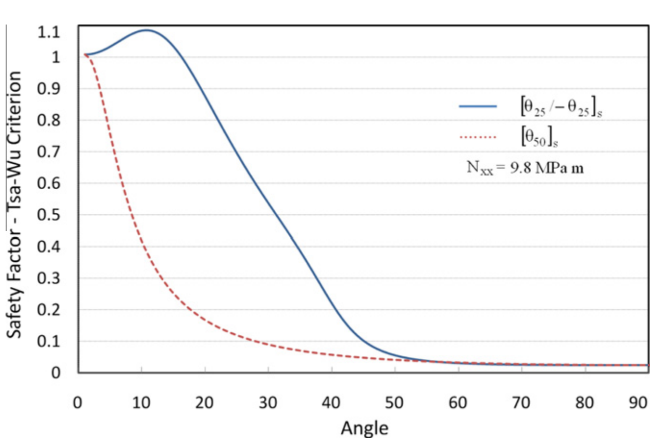

Fig. 5. Safety factor calculated using the Tsai–Wu criterion for a laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for a range of fiber orientation angles, θ.

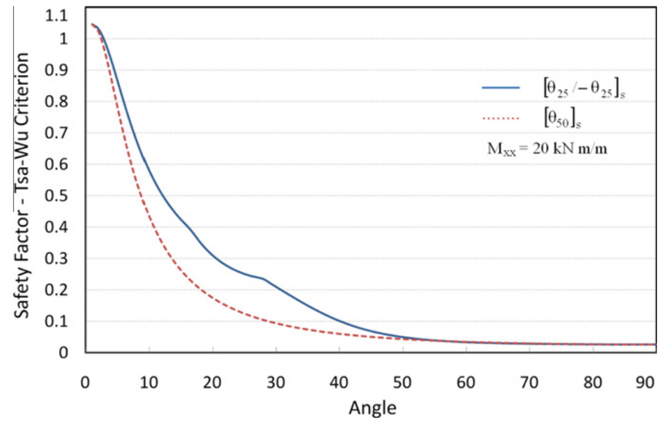

Fig. 6. Safety factor calculated using the Tsai–Wu criterion for a laminate subjected to one component of bending moment (only $M_{xx} \neq 0$) for a range of fiber orientation angles, θ.

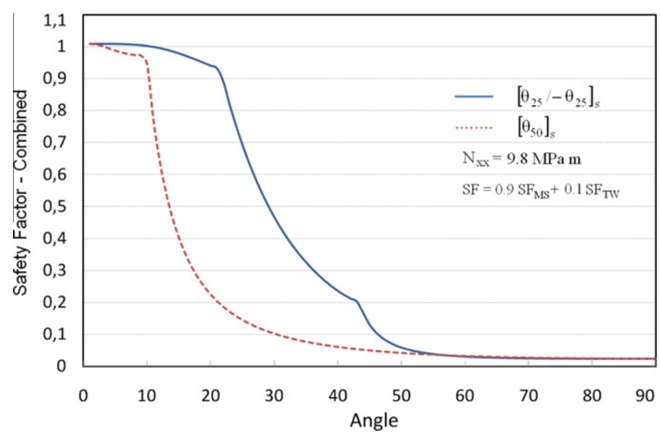

Fig. 7. Safety factor calculated using both criteria for a unidirectional laminate and a balanced laminate subjected to uniaxial loading (only $N_{xx} \neq 0$) for a range of fiber orientation angles, θ.

3. Methodology

3.1. Formulation of the objective function

When the load on a laminate is increased, only some of the plies may fail while the rest of the plies continue to resist the applied load. If the load is further increased, progressively more and more damage is induced until the whole laminate fails. In this study failure of a single ply is considered as signaling inception of the failure of the whole structure. Accordingly, the first-ply failure approach is adopted in the design optimization and safety of each lamina in a laminate design generated during the optimization process is checked using the Tsai–Wu and the maximum stress failure criteria. Failure is predicted if one of the inequalities in Eqs. (13) and (16) is not satisfied for any one of the laminae. In that case, a penalty value is calculated and added to the cost function. The overall cost function may then be expressed as

$$F = 2t_o \sum_{k=1}^{m} n_k + w_1 P_{MS} + w_2 P_{TW} - w_1 \mathrm{SF}_{MS} - w_2 \mathrm{SF}_{TW} \quad (22)$$

where the first term represents the total thickness of the composite structure as given in Eq. (2); n_k is the number of plies in the kth lamina, in which the orientation angle is θ_k; m is the total number of laminae below or above the mid-plane of the laminate; the second and the third terms represent the penalty values introduced to increase the value of the objective function for designs for which failure is predicted and thus to restrict the search to the feasible design space; P_{MS} and P_{TW} are penalty values calculated based on the maximum stress criterion and the Tsai–Wu criterion, respectively. SF_{MS} and SF_{TW} are equal to the safety factors according to the maximum stress and the Tsai–Wu criteria, respectively, if they are greater than 1.0, otherwise these terms are zero; w_i are suitable coefficients. Following the trends observed in Figs. 3–6, if the in-plane loading is dominant, the weight of the terms associated with the maximum stress criterion, w_1, is taken to be larger; otherwise w_2 is larger. Their values are chosen such that the combined safety factor reflects the experimentally observed trend. The values of 0.2 and 0.02 for w_1 and w_2 respectively, are expected to give the correct trend for in-plane loading cases as Fig. 7 implies. Because the failure envelope of the maximum stress criterion does not conform to the expected trend in bending, w_1 is taken as 0.0001, while w_2 is taken as 0.1 if the bending load is dominant.

The reason that the objective is reduced for safe designs is that there may be many feasible designs with the same minimum thickness. Of these designs, the optimum is defined as the one with the largest failure load. Accordingly, the objective function is linearly reduced in proportion to the failure margin as in Ref. [48]. As another approach, the margins to initial failure were maximized for the minimum feasible number of laminae in another study [49].

If a principal stress component exceeds its respective strength for a lamina, the maximum stress criterion is violated as indicated in Eq. (13) and a penalty value is calculated as in the following:

$$P_X^k = \begin{cases} 0 & \text{if } SF_X^k \geqslant 1.0 \\ \left(1/SF_X^k\right) - 1 & \text{if } SF_X^k < 1.0 \end{cases}$$
$$P_Y^k = \begin{cases} 0 & \text{if } SF_Y^k \geqslant 1.0 \\ \left(1/SF_Y^k\right) - 1 & \text{if } SF_Y^k < 1.0 \end{cases} \quad (23)$$
$$P_S^k = \begin{cases} 0 & \text{if } SF_S^k \geqslant 1.0 \\ \left(1/SF_S^k\right) - 1 & \text{if } SF_S^k < 1.0 \end{cases}$$

where P_X^k, P_Y^k, and P_S^k are the penalty values for the kth lamina associated with the failures due to loading along fiber direction, transverse to it, and shear loading, respectively. The total penalty value for the laminate due to the violation of the maximum stress criterion is then calculated by summing up the penalty values calculated for each lamina.

$$P_{MS} = \sum_{k=1}^{m} P_X^k + P_Y^k + P_S^k \quad (24)$$

On the other hand, if the Tsai–Wu criterion is violated for the kth lamina, the penalty value is calculated as

$$P_{TW}^k = \begin{cases} 0 & \text{if } SF_{TW}^k \geqslant 1.0 \\ \left(1/SF_{TW}^k\right) - 1 & \text{if } SF_{TW}^k < 1.0 \end{cases} \quad (25)$$

The total penalty value for the laminate due to the violation of the Tsai–Wu criterion is then found by summing up the penalty values calculated for each lamina.

$$P_{TW} = \sum_{k=1}^{m} P_{TW}^k \quad (26)$$

3.2. Optimization procedure

In order to search for the globally optimum laminate designs, a variant of simulated annealing (SA) algorithm is proposed. This is similar to the direct search simulated annealing (DSA) [45] in that a set of current configurations rather than a single current configuration is maintained during the optimization process unlike ordinary SA. Accordingly, unlike the standard SA algorithm, where only the neighborhood of a single point is searched, the neighborhood of all the current points in the set is searched. This feature resembles population in genetic algorithms. If a newly generated configuration is accepted, it replaces a current configuration other than the best configuration; thus the best current configuration is not lost. In this way, one of the most important drawbacks of the standard SA is avoided, where many good current solutions are replaced by worse solutions especially at the early stages of the optimization process.

At the start of the optimization process, N number of initial configurations are randomly created within the design domain through a random selection of values for the design variables. N is equal to $8(2m + 1)$, where $2m$ is the number of design variables as mentioned before. The design variables are the number of plies in the kth lamina, n_k, and the orientation angle of the fibers in these plies, θ_k. A number among $(0, 1, 2, \ldots, 20)$ is randomly chosen for n_k, and among $(-90, -90 + \phi, \ldots, -2\phi, -\phi, 0, \phi, 2\phi, \ldots, 90 - \phi, 90)$ for θ_k. Here ϕ is the interval between consecutive angles, which may be 1°, 5°, 10°, 15°, 30°, 45°, or 90°. If zero ply number is chosen, this means that no material exists for the respective lamina and this lamina does not contribute to the load carrying capacity of the laminate. After the initial laminate configurations are randomly chosen, their objective functions are calculated. SA requires random generation of a new configuration in each iteration. For this purpose, first, one of the current configurations is randomly chosen. Then, random differences are introduced to the ply numbers and fiber angles.

$$\begin{aligned} n'_k &= n_k + r_1 \Delta n_{\max} \\ \theta'_k &= \theta_k + r_2 \Delta\theta_{\max} \end{aligned} \quad (27)$$

Here n_k and θ_k are the ply number and fiber angle in the kth lamina of the randomly chosen current configuration; n'_k and θ'_k are the ply number and fiber angle of the newly generated configuration; r_i are randomly chosen real numbers within $[-1, 1]$; $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are the maximum variations that may be introduced to n_k and θ_k to generate a new lay-up configuration. The lower limit for n'_k is zero; if n'_k is negative, a new random number, r_1, is generated. There is no upper limit for n'_k. The lower and upper limits for θ'_k are −90° and 90°, respectively. If a number greater than 90 is generated for θ'_k, 180 is subtracted from this number. If it is less than −90, 180 is added. The values of n'_k and θ'_k are then rounded to the nearest discrete value. In SA, acceptability of a newly generated trial configuration, A_t, depends on its objective function value, f_t, which is calculated by

$$A_t = \begin{cases} 1 & \text{if } f_t \leqslant f_h \\ \exp((f_h - f_t)/T_j) & \text{if } f_t > f_h \end{cases} \quad (28)$$

Here f_h is the cost of one of the high-cost current configurations. According to Eq. (28), every new design having a cost lower than f_h is accepted. But, if the cost is higher, the trial configuration may be accepted depending on the value of A_t. If it is greater than a randomly generated number, P_r, the trial configuration is accepted, otherwise it is rejected. If the trial design is accepted, it replaces one of the inferior configurations. Iterations during which the value of the temperature (or control) parameter, T_j, is kept constant are called jth Markov chain (or inner loop). After a certain number of iterations, the temperature parameter, T, is reduced, a new inner loop begins. As Eq. (28) implies, when T is decreased, the probability that a configuration with a higher cost is accepted becomes lower. At the start, the control parameter is so large that almost any arbitrary configuration is accepted. At low values of temperature parameter, acceptability becomes low; thus, acceptance of inferior configurations is unlikely, just as the atoms become stable, and do not tend to change their arrangements at low temperatures.

In DSA, if a configuration generated during iterations is accepted, it replaces the worst current configuration. This is the one of the drawbacks of DSA that becomes especially apparent with a high number of design variables. For instance, if one optimizes a laminate with 16 distinct lamina angles and thicknesses using DSA, the number of current configurations will be $8(2 \times 16 + 1) = 264$. After the start of the optimization process, the current configurations quickly gather around local minimums except the worst one, which is frequently updated by higher-cost configurations at high temperatures. Because new configurations are generated in the neighborhood of the current ones, search becomes restricted to a small portion of the feasible domain except for the trials in which the new configuration is generated around the worst current configuration, which are very few. In order to remedy this, a different replacement scheme is adopted. The current configurations are ordered with respect to their objective function value. If a new configuration is accepted, it replaces a current configuration randomly chosen among $(2m + 1)$ worst configurations instead of the worst one. Thus, $7(2m + 1)$ of the current configurations having low cost remain in the set unless better ones are found through iterations. The others, on the other hand, may be replaced by higher cost configurations with a probability depending on the temperature parameter. In this way, $(2m + 1)$ number of configurations become dispersed in the feasible domain at high temperatures and thus a thorough search of the feasible domain can be achieved. In calculating the acceptability using Eq. (28), the objective function value of the best of the worst $(2m + 1)$ current configurations is used for f_h.

In order to find the globally optimal design, one should be able to search a large solution domain. For this reason, instead of giving small perturbations to the current configuration to obtain a new configuration in its near neighborhood, one should allow a large variance in the current configurations. For this reason, the magnitudes of $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are taken as 15 and 50, respectively. This means that the neighborhood of a current configuration where a new configuration is generated is initially quite large. This can also be considered as a logical consequence of simulating the physical annealing process, where mobility of atoms is large at high temperatures, and thus the probability that atoms may form a quite different configuration is high. Also, as in the physical process, where the mobility of atoms decreases as the temperature is lowered, variations in n_k and θ_k are also reduced as temperature parameter is decreased; but the reduction scheme does not directly depend on the level of temperature. If no improvement is obtained in the worst of the best $7(2m + 1)$ current configurations during a Markow chain, $\Delta n_{\max}$ and $\Delta\theta_{\max}$ are reduced by multiplying with a factor to make the searched region smaller and thus increase the likelihood of finding a better design.

The temperature parameter, T, controls the convergence of the optimization process just like the temperature controls microstructural changes in the physical annealing process. At the initial stages of the optimization, temperature should be high enough for the algorithm to accept almost any arbitrarily generated configuration regardless of the objective function value. At the final stages, it should take such low values that a new configuration that is worse than the current configurations is almost never accepted. These correspond to the melting and freezing temperatures in the physical annealing process, respectively. The reduction scheme is as follows

$$T_j = \alpha_j T_{j-1} \tag{29}$$

where T_j and T_{j-1} are the values of the temperature parameter in the jth and $(j-1)$th Markow chains, respectively, and α_j is temperature reduction factor. The other parameter controlling convergence is the maximum variation in the optimization variables, $\Delta n_{\max}$ or $\Delta\theta_{\max}$. At the beginning, large variations in the optimization variables are allowed so that the feasible domain is thoroughly searched for optimum designs. Towards the end of the optimization process, only close neighborhood of the current configurations is searched in order to exactly determine the value of the optimal point. As mentioned before the reduction scheme in $\Delta\theta_{\max}$ depends on whether improvement is achieved or not in a Markow chain. If a configuration is not found that is better than at least one of the best $7(2m + 1)$ current configurations, $\Delta\theta_{\max}$ is reduced as

$$\Delta\theta'_{\max} = c_1\Delta\theta_{\max} \quad \text{and} \quad \Delta n'_{\max} = c_2\Delta n_{\max} \tag{30}$$

The values of c_1 and c_2 depend on the dimension of the problem; c_1 is equal to 0.9 if the number of optimization variables is small; $c_1 = 0.999$ if the dimension of the problem is large. In this way, more time is given to thoroughly search the feasible domain for difficult problems. c_2 is chosen such that $\Delta\theta_{\max}$ and $\Delta n_{\max}$ reaches their minimum value, which is ϕ and 1.0, at the same time. Here, there is a similarity to the physical annealing process. Reduction in the temperature should be slow enough to allow time-dependent micro-structural changes to occur and to reach equilibrium. In simulated annealing, non-improvement in the current set implies that further changes are unlikely with the current values of $\Delta\theta_{\max}$ or $\Delta n_{\max}$. For this reason, a reduction scheme different from SA or DSA algorithms is adopted for the temperature parameter. Instead of reducing T_j by a specific ratio, it is made dependent on the reduction in $\Delta\theta_{\max}$ or $\Delta n_{\max}$, because it is a better indication of equilibrium at a specific temperature level. The temperature reduction factor, α_j in Eq. (29), is calculated as

$$\alpha_j = \begin{cases} \alpha_{\max} & \text{if } L_a^j/L^j < [(\Delta\theta_{\max})/(\Delta\theta_{i\max})]^2 + 0.01 \\ \alpha_{\min} & \text{else} \end{cases} \tag{31}$$

where L^j is the number of trials executed in the jth Markow chain and L_a^j is the number of accepted configurations. Accordingly, L_a^j/L^j ratio is a measure of acceptability in a Markow chain. $\Delta\theta_{i\max}$ is the initial maximum variation in the lamina angles. α_{max} and $\alpha_{\min}$ are taken to be 0.9999 and 0.9. Through the use of this equation, acceptability of Markow chains, L_a^j/L^j, is correlated to the ratio of the current and initial maximum variance, $\Delta\theta_{\max}/\Delta\theta_{i\max}$. When the acceptability is higher, temperature parameter is reduced at a faster rate, $\alpha_{\min}$; otherwise at a slower rate, $\alpha_{\max}$. At the initial stages of the optimization, the right hand side of the inequality is close to 1.0; accordingly, the temperature level is kept high such that the ratio of the number of accepted trials to the total number of trials is close to 1.0. When $\Delta\theta_{\max}$ approaches zero at the end of the optimization process, temperature parameter also approaches zero; acceptability of a new configuration, A_t in Eq. (28), then comes close to zero. Iterations are continued until the difference between the values of the best and the worst current configurations becomes small.

One of the disadvantages of the global search algorithms is that many parameters should be tuned for the specific problem at hand. Otherwise, expected performance cannot be obtained. Finding suitable values, on the other hand, requires extensive numerical experiments. In this respect, the application of the present algorithm to a different problem is much easier. Suitable values for most of the parameters can be found without difficulty. The initial value of the temperature parameter, T, in Eq. (28) is chosen so high that almost all of the randomly generated configurations are accepted; but unlike the other variants choosing a much higher than necessary has little effect on the number iterations executed during the optimization process. This is because the reduction in the temperature parameter is made dependent on the reduction in $\Delta\theta_{\max}$ through Eq. (31). If the temperature parameter has a very high value, it is quickly reduced to the levels that will yield the required acceptability ratio, L_a^j/L^j. The initial value of the maximum variation that may be introduced to the optimization variables, $\Delta\theta_{i\max}$, is another parameter affecting convergence. Its value is chosen based on the extent of the solution domain. Its value should be chosen as high as possible in order to search the solution domain thoroughly. For the present problem, the initial value of $\Delta\theta_{\max}$ is chosen as 50°, knowing that the upper and lower limits of θ are 90° and −90°. The value of factor c_1 in Eq. (30) depends on the difficultly of the problem. The higher the number, the more iterations are executed. If the number of variables is four or less than four, 0.9 is a suitable value. If the number of variables is 16 or more, a much slower reduction in the maximum variation is required; a number as high as 0.999 permits a thorough search of the solution domain. The only parameter that requires numerical experiments to determine its value is c_1.

4. Numerical results and discussions

The numerical results were obtained for a graphite/epoxy material, T300/5308, with the following material properties: E_{11} = 40.91 GPa, E_{22} = 9.88 GPa, G_{12} = 2.84 GPa, ν_{12} = 0.292, X_T = 779 MPa, X_C = −1134 MPa, Y_T = 19 MPa, Y_C = −131 MPa, S = 75 MPa. Thickness of the plies is 0.127 mm.

4.1. Dependence of the optimum designs on the failure criterion

The results of the optimization process depend on the failure criterion. Table 1 shows the optimal designs obtained by applying the aforementioned optimization procedure using different failure criteria. The loading is uniaxial ($N_{xx} = 10 \times 10^6$ N/m) and two distinct fiber orientations are used. The interval between the angles is taken to be 1°. If only the Tsai–Wu criterion is used, the optimal design is almost balanced and the angle imparting the highest strength is predicted around ±10°, conforming to the trend shown in Fig. 5, which is obtained using only a single angular parameter, θ. According to the maximum stress criterion, however, this configuration is unsafe. If only the maximum stress criterion is used, the optimal laminate is unidirectional with 5° fiber orientation angle, conforming to the trend shown in Fig. 3. According to the Tsai–Wu criterion, however, this design is highly nonconservative. These results imply that relying on just one failure criterion may lead to spurious optimal designs. If the Tsai–Wu and maximum stress criteria are used together, the optimal lay-up design agrees with the empirical observations; i.e. a laminate under uniaxial loading is strongest if all the fibers are aligned along the load direction. Table 2 shows the optimal designs of laminates under bending loading for various uses of failure criteria. The optimum laminates are not unidirectional according to both criteria. For $[0_{43}]_s$, the safety factor is 1.0325. The optimum designs, on the other hand, have a higher safety factor. When both criteria are used, the optimal design has a safety factor (1.1124) very close to that of the optimal design found by using only the Tsai–Wu criterion; but the safety factor based on the maximum stress criterion is larger (1.0123 in comparison to 1.0115) even though its weight, w_1 Eq. (22), is very small. One may conclude that use of the two failure criteria together shows a potential in composite optimization.

4.2. Optimum designs for different numbers of distinct laminae

For some out-of-plane loading cases, the optimal lay-ups having minimum thickness were obtained using the Tsai–Wu and maximum stress criteria together in order to see the effectiveness of the optimization algorithm proposed in this study. A range of values were tried for the number of distinct laminae. Tables 3–6 show the optimum angles, the number of plies oriented along these angles, and the total number of plies for laminates subjected to various out-of-plane loads. For the cases presented in the tables, in-plane loads are zero, $N_{xx} = N_{yy} = N_{xy} = 0$ N/m, unless otherwise stated. Since stacking sequence affects the laminate response for out-of-plane loading, only the adjacent plies are shown by a single symbol, e.g. $[90/90/0_2/0/90_3/0]_s$ may be shown as $[90_2/0_3/90_3/0]_s$, but not as $[90_5/0_4]_s$, because they lead to different stress and strain states under the same out-of-plane loading. Furthermore, if the optimum number of plies in a lamina is found to be zero, it is not shown in the results because no material exists in that lamina.

For biaxial bending (M_{xx} = 15, M_{yy} = 15, M_{xy} = 0), quite a number of multiple globally optimum lay-up configurations were found as shown in Table 4. For two distinct fiber angles, the optimization algorithm found $[-90_{27}/0_{150}]_s, [-89_{27}/1_{150}]_s, \ldots, [-1_{27}/89_{150}]_s, [0_{27}/90_{150}]_s, [1_{27}/-89_{150}]_s, \ldots, [90_{27}/0_{150}]_s$ as optimal configurations; all of them had the same objective function value. This means that the strength of a laminate having $[-90_{27}/0_{150}]_s$ lay-up is the same for all biaxial bending loads having equal magnitude applied along any arbitrary x–y directions. For this loading case, tensile stresses transverse to the fibers are critical. Because for all these lay-ups, $[\theta_{27}/90+\theta_{150}]_s$, transverse tensile stress in the outermost ply is the same, they have the same safety factor, and thus the same

Table 1
Dependence of the optimal designs on the chosen failure criteria for the loading $N_{xx} = 10 \times 10^6$, $N_{yy} = N_{xy} = 0$ N/m, and for two distinct fiber angles. $M_{xx} = M_{yy} = M_{xy} = 0$ Nm/m.

Failure criteria used to check feasibility	Optimal lay-up	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. str.
Only Tsai–Wu criterion	$[-9_{25}/10_{22}]_s$	94	1.0007	0.9142
Only max stress criterion	$[5_{51}]_s$	102	0.6688	1.0168
Both Tsai–Wu and max. stress	$[0_{51}]_s$	102	1.0091	1.0091

Table 2
Dependence of the optimal designs on the chosen failure criteria for the loading M_{xx} = 15, $M_{yy} = M_{xy}$ = 0 kNm/m, and for two distinct fiber angles. $N_{xx} = N_{yy} = N_{xy}$ = 0 N/m.

Failure criteria used to check feasibility	Optimal lay-up	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. str. stress
Only Tsai–Wu criterion	$[3_{15}/-10_{28}]_s$	86	1.1124	1.0115
Only max stress criterion	$[-12_4/4_{39}]_s$	86	1.0102	1.0563
Both Tsai–Wu and max. stress	$[2_{21}/-16_{22}]_s$	86	1.1124	1.0123

Table 3
The optimum lay-ups for the loading M_{xx} = 15, $M_{yy} = M_{xy}$ = 0 kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
1	$[0_{43}]_s$	86	1.0325	1.0325
2	$[2_{21}/-16_{22}]_s$	86	1.1124	1.0123
4	$[0_2/-6_9/7_8/6_{24}]_s$	86	1.1164	1.0122
8	$[0_2/-6_3/-7_5/6_{31}/-14_2]_s$	86	1.1165	1.0122
16	$[0/-5_2/-6_4/7_7/6_4/-5_{12}/7_{12}/26_1]_s$	86	1.1165	1.0122

Table 4
The optimum lay-ups for the loading $M_{xx} = 15$, $M_{yy} = 15$, $M_{xy} = 0$ kNm/m for various numbers of distinct fiber angles. $\alpha, \theta, \beta, \ldots$ are arbitrary integer angles between −90° and 90°.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
1	$[\theta_{272}]_s : [90_{272}]_s, [89_{272}]_s, [88_{272}]_s, \ldots$	544	1.0073	1.0076
2	$[\theta_{27}/90+\theta_{150}]_s : [-90_{27}/0_{150}]_s, [-89_{27}/1_{150}]_s, \ldots$	354	1.0052	1.0115
4	$[\theta_{10}/\theta+29_{11}/\theta-45_{16}/\theta+87_{138}]_s, [\theta_{10}/\theta-29_{11}/\theta+45_{16}/\theta-87_{138}]$	350	1.0113	1.0171
8	$[-39_4/-57_5/-66_6/-6_5/-82_6/10_8/74_{12}/37_{127}]_s$	346	1.0001	1.0030
16	$[-27_5/-45_4/-2_4/4_5/11_5/-72_5/25_3/30_3/-90_5/78_{33}/71_{23}/49_{26}/79_{38}/13_{14}]_s$	346	1.0024	1.0065
50	$[-39_3/-52_3/-19_3/-14_4/-69_3/-4_4/-79_3/-83_4/-89_3/16_3/81_3/27_4/66_2/68_3/36_{23}/66_{18}/61_3/38_{11}/44_4/67_{10}/68/44_2/60_4/56_{10}/37_5/36_2/39_{11}/54_8/39_2/17_3/19_3/-76/53_2/58_3/-49/7_1]_s$	346	1.0047	1.0080

Table 5
The optimum lay-ups for the loading $M_{xx} = M_{yy} = M_{xy} = 15$ kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
1	$[45_{60}]_s$	120	1.0052	1.0052
2	$[38_{13}/52_{47}]_s, [52_{13}/38_{47}]_s$	120	1.0523	1.0009
4	$[53_9/39_{22}/53_6/36_{23}]_s$	120	1.0563	1.0000
8	$[37_4/38_6/52_{25}/38_7/40_3/44_4/43_{11}]_s$	120	1.0564	1.0000
16	$[53_7/39/51_5/38_{10}/38_9/38_{11}/45/38_5/52_3/36_4/56/40_3]_s$	120	1.0564	1.0000
50	$[53_7/38_{10}/50/38_5/40/50_3/51_3/52_3/39_4/52_3/38_4/37_5/56_3/52_3/43/46_2/38/36]$	120	1.0564	1.0000

Table 6
The optimum lay-ups for the loading $M_{xx} = M_{yy} = 0$, $M_{xy} = 15$ kNm/m for various numbers of distinct fiber angles.

Number of distinct fiber angles	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
1	$[0_{137}]_s, [90_{137}]_s$	274	1.0091	1.0091
2	$[6_{63}/-77_{71}]_s, [84_{63}/-13_{71}]_s$	268	1.0127	1.0144
4	$[-82_{35}/-78_{19}/14_{38}/27_{38}]_s$	260	1.0070	1.0079
8	$[8_{30}/-79_{17}/14_{11}/16_{10}/-72_{12}/-68_{11}/29_{11}/-42_{25}]_s$	254	1.0111	1.0114
16	$[-82_{16}/10_{16}/12_{15}/-76_{10}/16_5/-73_{10}/20_6/-68_6/-65_6/-61_4/-57_4/39_3/-35_{15}/54_2/-36_8]_s$	252	1.0066	1.0067
50	$[80_{18}/78_8/77_{17}/-14_{16}/-17_8/-19_6/68_5/-22_5/-25_5/-28_3/-30_2/-32_2/-34_3/-39_3/-48/-54_7/-56_{14}/-47/-54/17]_s$	252	1.0161	1.0163

objective function value. Because, the material is weakest for tensile loads applied transverse to the fibers, the optimum laminates have thickness much higher than the one subjected to only one component of bending moment ($M_{xx} = 15$, $M_{yy} = M_{xy} = 0$ kNm/m). It should be noted that such thick laminates are not used in real applications. If the out-of-plane loads are large, stiffened laminates are preferred. However, in order to test the effectiveness of the algorithm, thick laminates that allowed innumerous different configurations were optimized in this study.

By increasing the number of distinct fiber angles, better lay-up designs, i.e. either thinner laminates or laminates with a larger safety factor, can be obtained. One should also note that when a larger design domain is chosen, the number of local optimums and the complexity of the design domain may dramatically increase. In that case, a reliable global search algorithm should be used to be able to locate the globally optimum design(s). With 16-distinct laminae, one-degree interval between possible fiber angles, and lamina thicknesses up to 30–40 plies or even more, $180^{16} \cdot 30^{16} \cong 5 \times 10^{59}$ number of distinct lay-up configurations exist. With 50-distinct laminae, this number exceeds 10^{186}. In addition to multiple global optimums, numerous near global optimum designs exist. Obtaining an improved result with such a large solution domain attests the reliability of the search algorithm proposed in this study. Improvement is more pronounced for the biaxial bending (Table 4) and pure twisting loads (Table 6). Even for cases in which improvement is insignificant, many alternative designs can be obtained by enlarging the design domain.

4.3. Optimum designs for different intervals between consecutive fiber angles

Table 7 shows the optimal designs obtained using various intervals between the consecutive fiber orientation angles for the loading $M_{xx} = M_{yy} = 0$, $M_{xy} = 15$ kNm/m. As the results indicate, better designs can be obtained within a design domain enlarged by choosing a smaller interval. For other loading cases, choosing a large interval may lead to gravely inferior designs. For this reason, one should choose the minimum interval that the manufacturing method allows.

4.4. Optimum designs for different biaxial bending loading cases

Table 8 shows the optimal laminate designs for various biaxial bending loading cases obtained using two distinct fiber angles. For the loading case $M_{xx} = 10$, $M_{yy} = 5$, $M_{xy} = 0$ kNm/m, the optimal lay-up is $[31_{20}/-43_{88}]_s$ with 216-ply thickness. When M_{xx} is increased to 20 kNm/m, strangely the optimal laminate becomes thinner. This counter intuitive optimal design can be explained by considering the differences in the stress states. When M_{xx} is increased to 20 kNm/m and the laminate design is changed to $[30_{19}/-33_{80}]_s$, σ_{11} increases from 71.4 MPa to 166.6 MPa and ε_{11} increases from 0.16×10^{-2} to 0.40×10^{-2}; ε_{22}, on the other hand, decreases from 0.14×10^{-2} to 0.05×10^{-2} due to Poisson's effect. The stress transverse to the fibers then decreases from 18.6 MPa to 16.2 MPa, while the other principal stresses (shear stress and

Table 7
The optimum lay-ups for the loading $M_{xx} = M_{yy} = 0, M_{xy} = 15$ kNm/m, and for various intervals between fiber angles.

Interval between orientation angles	Optimum lay-up sequences	Total number of plies	Safety factor f or Tsai–Wu	Safety factor for max. stress
90°	$[0_i, 90_{137-i}]_s$: $[90_{137}]_s$, $[0_1, 90_{136}]_s$, ... $[90_i, 0_{137-i}]_s$: $[0_{137}]_s$, $[90_1, 0_{136}]_s$,...	274	1.0091	1.0091
45°	The same as above	274	1.0091	1.0091
30°	$[0_{106}/-30_{31}]_s$, $[0_{106}/30_{31}]_s$, $[90_{106}/60_{31}]_s$, $[90_{106}/-60_{31}]_s$,	274	1.0130	1.0356
15°	$[0_{86}/15_{49}]_s$, $[0_{86}/-15_{49}]_s$, $[90_{86}/75_{49}]_s$, $[90_{86}/-75_{49}]_s$	270	1.0068	1.0144
10°	$[0_{70}/10_{65}]_s$, $[0_{70}/-10_{65}]_s$, $[90_{70}/80_{65}]_s$, $[90_{70}/-80_{65}]_s$	270	1.0128	1.0156
5°	$[85_{74}/-15_{60}]_s$, $[-85_{74}/15_{60}]_s$, $[5_{74}/-75_{60}]_s$, $[-5_{74}/75_{60}]_s$	268	1.0081	1.0125
1°	$[6_{63}/-77_{71}]_s$, $[84_{63}/-13_{71}]_s$	268	1.0127	1.0144
Continuous fiber angles	$[6.32_{60}/-77.06_{73}]_s$, $[83.68_{60}/-12.94_{73}]_s$	266	1.0015	1.0018

Table 8
The optimum lay-ups obtained using two distinct fiber angles for various loading cases.

Loading: M_{xx}/ M_{yy}/M_{xy} (kNm/m)	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
5/5/0	$[\theta_{16}/\theta-90_{87}]_s$: $[0_{16}/90_{87}]_s$, $[1_{16}/-89_{87}]_s$, ...	206	1.0182	1.0245
10/5/0	$[31_{20}/-43_{88}]_s$, $[-31_{20}/43_{88}]_s$	216	1.0044	1.0195
20/5/0	$[30_{19}/-33_{80}]_s$, $[-30_{19}/33_{80}]_s$	198	1.0197	1.1698
30/5/0	$[26_{19}/-30_{74}]_s$, $[-26_{19}/30_{74}]_s$	186	1.0238	1.5369
40/5/0	$[25_{18}/-27_{74}]_s$, $[-25_{18}/27_{74}]_s$	184	1.0060	1.4522
50/5/0	$[20_{21}/-26_{74}]_s$, $[-20_{21}/26_{74}]_s$	190	1.0105	1.3196

normal stress along the fiber direction) increase. This means that when M_{xx} is increased from 10 kNm/m to 20 kNm/m, the more damaging stress component decreases in the optimal designs, while the stress components against which the laminate is much stronger increase. Because, the transverse tensile stresses are critical, a thinner laminate could carry a larger load. If M_{xx} is increased up to 40 kNm/m, the same trend continues. However, if it is increased to 50 kNm/m, a thicker laminate is required.

4.5. Optimum designs with contiguity and angle difference constraints

In laminate design, a limit is set to the number of contiguous plies of the same orientation to prevent matrix damage propagation and thus avoid large matrix cracks; the maximum thickness is experimentally determined and it is usually taken as four plies for CFRP laminates [50]. Besides, in order to reduce edge delaminations, the difference between the angles of two consecutive laminae should not exceed 45° [50]. These requirements can be taken into account in the present optimization procedure by imposing constraints and adding penalty values to the objective function whenever these constraints are violated. For this purpose, whenever a new configuration is generated, these requirements are checked; if the difference between the fiber angles of two consecutive laminae, $\Delta\theta$, is greater 45°, a penalty value equal to $\Delta\theta - 45$ is added. If the number of plies in a lamina, n_k, is larger than four, a penalty value equal to $10(n_k - 4)$ is added. If the difference between the fiber angles of consecutive laminae is less than 5°, the sum of the number of plies in those laminae is also constrained not to exceed four. With these constraints, results were generated for 60–70 distinct lamina angles. Because a lamina may not contain more than four plies, the initial value of the maximum variation for thickness, $\Delta n_{\max}$, was taken as three. Table 9 presents the results for different loading cases. The laminate designs satisfying these constraints have the same thicknesses as the ones obtained without imposing them (Tables 4–6) except for the loading case of pure twisting moment. This may be because there are many near global optimum designs and some of them satisfy these constraints. For the pure twisting case, however, the optimum laminates satisfying the constraints are about 10% thicker.

4.6. Comparative performances of SA algorithms

In order to see the relative performance of the SA algorithm proposed in the present study, a comparative study was conducted. The performance of a global search algorithm is defined as the number of times the global optimum is found divided by the number of functional evaluations. This is called the reliability index, *Rel*.

$$Rel = \frac{\text{No. of global optimums}}{\text{No. of functional evaluation}} \tag{32}$$

Four different SA algorithms and a local search algorithm are considered: the present one, the classical simulated annealing algorithm (SA) [44], the direct search simulated annealing (DSA) [45], the modified direct search simulated annealing (MDSA) proposed in the previous study conducted by the authors [43], and Nelder & Mead, a deterministic local search algorithm. The main differences between the present method and MDSA are the replacement scheme of accepted configurations, and the reduction scheme for the temperature parameter (Eq. (31)). The main difference between DSA and MDSA is the scheme used to generate random configurations (Eq. (27)). In MDSA, the neighborhood of the current point in which a new configuration is randomly chosen becomes narrower and narrower throughout the optimization process. This is achieved by decreasing $\Delta n_{\max}$ and $\Delta\,\theta_{\max}$ in Eq. (27) according to the procedure explained previously. The main difference between DSA and the classical SA is that a set of current configurations is used in DSA instead of a single current configuration as in SA. Reliability indices are normalized by dividing their value by the reliability index of the present search algorithm, Rel_c, to obtain what is called performance index, *PI*.

$$PI = 100\frac{Rel}{Rel_c} \tag{33}$$

A laminate subjected to pure twisting moment, $M_{xy} = 10$ kNm/m, is considered for the comparative study. In order to calculate the value of the reliability index, optimization process is repeated more than 300 times using the respective search algorithm. The interval between fiber orientation angles is chosen as 1°. Different numbers for distinct laminae are tried. Especially when this num-

Table 9
The optimum laminate designs satisfying the contiguity constraint, i.e. no lamina may contain more than four plies, and the constraint that the difference between two consecutive laminae may not exceed 45°.

Loading: $M_{xx}/M_{yy}/M_{xy}$ (kNm/m)	Optimum lay-up sequences	Total number of plies	Safety factor for Tsai–Wu	Safety factor for max. stress
15/15/15	$[53_3/38/37_3/51_4/38_3/52_3/38_4/53_3/39_4/51_2/38_4/53_3/41_3/36_2/51_4/37_4/55/39_2/55_2/47/44/53/45_2]_s$	120	1.0564	1.0000
15/15/0	$[23_4/40_4/47_4/2_2/-10_4/-16_4/-11_2/-25_4/-31_4/-39_4/-84_4/-70_3/-85_4/-73_4/-81_4/-86_3/89_3/-80_4/88_4/-85_2/87_4/-87_4/86_4/-83_4/-59_4/-36/-54_3/-77_4/81_3/-63/-80_4/86_4/72/-88_2/-82_3/79_3/-89_4/-72_4/-89_3/-48_3/-26/-66_2/-57_4/-20_2/-63_4/-79_2/-75_2/-38_2/-67_4/90_4/-59_3/-47_4/-30_2/-14_2/-20_2]_s$	348	1.0107	1.0132
0/0/15	$[58_4/26_2/56_4/28/54_4/31_3/51_4/46_3/68/-67/-31_4/-64_2/-34/-35_3/-60_3/-40_4/-48_4/-43_3/-48_4/-42_4/-49_4/-43_4/-48_4/-43_4/-50_3/-42_4/-49_4/-43_3/-48_4/-42_3/-53_4/-45_2/-39_4/-49_4/-42_3/-54_3/-46_3/-40_3/-52/-68_3/-45_3/-81/73/-86/71/79_3/-66_2/75_2]_s$	280	1.0043	1.014

ber is high, numerous near global optimum designs are obtained. For example, if the laminate consists of 16 distinct laminae, the minimum thickness is obtained as 206 plies in 18% of the runs using the present method. The layup sequences of these laminate designs are found to be different having safety factors ranging from 1.001 to 1.012. In 80% of the runs, the minimum number of plies is 208; in 2% of the runs, it is 210. In calculating the reliability index, all of the near global optimums are considered as the global optimum and the number of runs in which the minimum thickness is found to be 206 is divided by the total number of iterations to calculate the reliability index. Similarly, the reliability indices and then performance indices of the other algorithms are calculated. Table 10 presents the results. The performance index of the present method is 100 by definition. As expected the performance of the present method is better than MDSA especially for problems with a high number of optimization variables. The performances of DSA and SA are rather poor. This indicates the importance of reducing the search domain during the optimization process. Table 10 also shows the relative performance of Nelder & Mead, which is applied to the problem with a multi-start approach. The initial values of the optimization variables are randomly chosen; the optimum point is then found according to the decision criteria of the search algorithm. Because the variables are continuous, the resulting optimum values of the lamina angles and thicknesses are rounded to the nearest permissible discrete value. The objective function value is then reevaluated using the discrete values. After repeating the optimization process many times (more than 10^5 times), performance indices are calculated. As Table 10 shows optimization with a local search algorithm is feasible only if the design variables are few. One may find the globally optimum design of laminates with two distinct lamina angles (or four design variables); but still the performance of Nelder & Mead algorithm is poor. With eight variables, one is unlikely to find the globally optimum design; with 16 or 32 variables, it is almost impossible. The reason for the low performance with a small number of optimization variables is the use of continuous optimization variables in the local optimization algorithm. When the optimum values of the variables are rounded to the nearest discrete values, either constraints are violated or objective function value increases.

4.7. Comparative performances of a genetic algorithm and the present SA algorithm

Lopez et al. [51] minimized the thickness of composite laminates subjected to static in-plane loads using a genetic algorithm. They compared the performances of their genetic algorithm and the one proposed by Le Riche and Haftka [52] using the performance criterion provided in that reference and found theirs better. In the performance criterion, a practical optimum is defined as the local optimum close to the global optimum by 0.1%. Then, reliability of the algorithm is defined as the probability that the algorithm finds a practical optimum in 6000 functional evaluations. This probability is calculated after conducting hundreds of independent runs; then the number of times that a practical optimum is found is divided by the number of runs. Table 11 presents the optimum lay-up sequences of laminates subjected to various in-plane loads and the reliabilities of the genetic algorithm used by Lopez et al. [51] and the present algorithm. The laminate design is $[0_{2q}/\pm45_r/90_{2s}]_s$, where q, r, and s are the integer variables used to minimize the thickness. This means that there are only three optimization variables. In order for the optimization process to converge in 6000 iterations, the value of factor c_1 in Eq. (30) was chosen as 0.82 and the initial value of the maximum variation for thickness, $\Delta n_{\max}$, was chosen as 12. As seen in the table, the reliability of the present algorithm is better for this problem whether Tsai–Wu or the maximum stress is used as the failure criterion; but the real difference is expected to manifest itself for problems with high numbers of optimization variables.

If the number of design variables is four, convergence to the optimum design takes about 10,000 functional evaluations using the present optimization algorithm. If the dimension of the problem is very large, it may take millions of iterations. Even in that case, optimization process takes only a few hours with the condition that smooth panels are optimized and the classical lamination theory is used in the structural analysis. It should be noted that a structure with a complex shape cannot be analyzed with an analytical method. If a numerical method like FEM is used, optimization of a large structure with many variables cannot be achieved using the present algorithm within a reasonable time with today's computational capabilities. For such applications, the following

Table 10
Comparison of the performances of various simulated annealing algorithms and a local search algorithm.

Number of distinct fiber angles	Minimum number of plies	Performance Index (PI)				
		Present SA	Modified DSA [43]	DSA [45]	SA [44]	Nelder and Mead
16	206	100	62	0	0	0
8	208	100	85	1	0	0
4	212	100	98	33	15	0.5
2	218	100	105	50	22	32

Table 11
Comparison of the reliabilities of the genetic algorithm used by Lopez et al. [51] and the present algorithm.

N_{xy} (N/mm)	Optimum lay-up sequences	Failure Criterion*	Safety factor	Reliability	
				Ref. [51]	Present
0	$[\pm45_{17}]_s$, $[0_{10}/\pm45_7/90_{10}]_s$, ...	MS	1.0455	1.00	1.00
0	The same	TW	1.1599	0.99	1.00
100	$[\pm45_{17}]_s$	MS	1.0245	1.00	1.00
100	The same	TW	1.1412	1.00	1.00
250	$[0_{18}//90_{16}]_s$,$[0_{16}//90_{18}]_s$	MS	1.0117	0.77	0.84
250	The same	TW	1.0161	0.32	0.59
500	$[\pm45_{18}]_s$	MS	1.0041	0.96	1.00
500	The same	TW	1.1243	1.00	1.00
1000	$[\pm45_{20}]_s$	MS	1.0209	0.96	1.00
1000	The same	TW	1.1302	0.41	1.00

* MS and TW denote the maximum stress criterion and Tsai–Wu criterion, respectively.

multilevel optimization procedure is suggested: First, the whole structure is analyzed using FEM and the stress states at critical locations are determined. Given the stress components at each layer, σ^k_{xx}, σ^k_{yy}, and τ^k_{xy}, the stress resultants, N_{ij}, M_{ij}, are calculated using Eqs. (6) and (7), Then, the lamina angles and thicknesses are optimized using the present optimization algorithm. When the laminate design is changed, stiffness properties of the structure and thus stress state also change. For this reason, the analysis of the whole structural is carried out again. This procedure is repeated until convergence.

5. Conclusions

In this study, a methodology is presented to optimize composite laminates subjected to both in-plane and out-of-plane loading for minimum thickness. A variant of the simulated annealing algorithm is proposed to search the globally optimum design(s). Many multiple global or near global optimums were found to exist. The algorithm proved to be more reliable in locating these designs in the benchmark tests compared to the other SA algorithms and a genetic algorithm recently proposed. The search algorithm yielded consistent and reliable results in all the optimization runs.

By increasing the number of distinct lamina angles and the range of values they may take, one obtains a larger design domain, i.e. more lay-up configurations become possible. In this way, the complexity of the design domain and the number of local minima greatly increase; but existence of a better global optimum becomes also more likely. In this study, up to fifty distinct lamina thicknesses and lamina angles with 1° angle increments were used as design variables to optimize laminates. To the knowledge of the authors, optimization with such a large solution domain was not attempted in the previous studies. With a larger domain, it was possible to obtain better optimum designs. Unlike in-plane loading, using two or three distinct lamina angles is not sufficient to obtain the best possible design for laminates under out-of-plane loading. For all the out-of-plane loading cases considered in this study, optimum lay-up configurations with 16-distinct laminae turned out to be better than the ones with 8-distinct laminae.

Results were also obtained by imposing contiguity constraint, i.e. no lamina may contain more than four plies, and the constraint that the difference between two consecutive laminae may not exceed 45°. In many of the loading cases, it was possible to find a near – global optimum design satisfying these constraints. In one case, a thicker laminate was required to satisfy them.

When the maximum stress or Tsai–Wu failure criterion is used individually, an optimization algorithm may lead to false optimal designs because of the particular features of their failure envelopes. On the other hand, when they are used together, false optimums may be avoided. For some cases, the best orientation angles according to one criterion may be quite unsafe according to the other. These designs can be avoided by using both criteria.

In some loading cases, the optimal designs can be counter intuitive. Sometimes, when one component of loading is increased, it is possible to find a thinner laminate design that can carry the applied load. Therefore, a design process for composite materials should not be based on intuition or experience.

If the available fiber orientations are scarce, quite inferior designs may be obtained. This is the case, if only 0°, ±45°, and 90° angles are allowed. For this reason, the interval between the consecutive angles should be selected as small as the manufacturing precision allows.

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