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Empirical Equations for the Prediction of PGA, PGV, and Spectral Accelerations in Europe, the Mediterranean Region, and the Middle East

Sinan Akkar
Middle East Technical University

Julian J. Bommer
Imperial College London

Online material: Digital data file of Table 1, the values of the coefficients for prediction of median pseudo-spectral accelerations and the associated standard deviations.

INTRODUCTION

The true performance of ground-motion prediction equations is often not fully appreciated until they are used in practice for seismic hazard analyses and applied to a wide range of scenarios and exceedance levels. This has been the case for equations published recently for the prediction of peak ground velocity (PGV), peak ground acceleration (PGA), and response spectral ordinates in Europe, the Middle East, and the Mediterranean (Akkar and Bommer 2007a,b). This paper presents an update that corrects the shortcomings identified in those equations, which are primarily, but not exclusively, related to the model for the ground-motion variability.

Strong-motion recording networks in Europe and the Middle East were first installed much later than in the United States and Japan but have grown considerably over the last four decades. The databanks of strong-motion data have grown in parallel with the accelerograph networks, and in addition to national collections there have been concerted efforts over more than two decades to develop and maintain a European database of associated metadata (*e.g.*, Ambraseys *et al.* 2004).

As the database of strong-motion records from Europe, the Mediterranean region, and the Middle East has expanded, there have been two distinct trends in terms of developing empirical ground-motion prediction equations (GMPEs): equations derived from a large dataset covering several countries, generally of moderate-to-high seismicity; and equations derived from local databanks for application within national borders. We refer to the former as pan-European models, noting that this is for expedience since the equations are really derived for southern Europe, the Maghreb (North Africa), and the active areas of the Middle East. The history of the development of both pan-European and national equations is discussed by Bommer *et al.* (2010), who also review studies that consider the arguments for and against the existence of consistent regional variations. The purpose of the present paper is not to revisit this discussion, since our premise is that there is no compelling evidence for regional variations in motions from moderate-to-large magnitude earthquakes, even if such differences are present for smaller events (*e.g.*, Douglas 2007; Stafford, Strasser, and Bommer 2008; Atkinson and Morrison 2009; Chiou *et al.*, forthcoming). We are of the view that for the purposes of seismic hazard analyses, the development of well-constrained equations applicable to the entire European region is desirable, even if these are then combined with local equations in hazard analyses. Certainly, well-constrained pan-European models are preferable to equations derived from sparse databanks of records that happen to have been recorded within a particular political boundary.

The first empirical equations for the prediction of response spectral ordinates in the European region were presented by Ambraseys *et al.* (1996). These equations were updated by Bommer *et al.* (2003), using exactly the same dataset, to include the influence of style-of-faulting as an additional explanatory variable, although this was done mainly for the purpose of investigating the effectiveness of an approximate approach to including style-of-faulting adjustments. An entirely new European GMPE for response spectral ordinates was presented by Ambraseys *et al.* (2005), using an expanded databank and with revised metadata.

Akkar and Bommer (2007a) presented a new GMPE, based on the same database as Ambraseys *et al.* (2005). This was not motivated by any perceived shortcoming with the Ambraseys *et al.* (2005) model but rather aimed to address additional requirements emerging in earthquake engineering. The primary motivation was to extend the range of response periods covered by

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the equations, since Ambraseys *et al.* (2005) only covered the range up to 2.5 seconds. Some emerging approaches to displacement-based seismic design, as well as the design of base-isolated structures, require spectral ordinates at longer periods as well as at damping values other than the ubiquitous 5% of critical. For this reason, the Akkar and Bommer (2007a) equations were derived to predict directly spectral displacements (SD); for completeness an equation was also derived for peak ground acceleration (PGA). On the basis of setting tolerable degrees of difference between the spectral ordinates of the filtered and raw accelerograms, criteria for the definition of the usable period range of the filtered records were established (Akkar and Bommer 2006). All of the accelerograms in the databank were reprocessed, and we selected an optimal low-cut filter for each record and employed the spectral ordinates only within the consequent usable range of response periods. This led us to conclude that the range of response periods could be extended to 4.0 seconds. Equations for spectral ordinates at five levels of damping were derived, following the finding of Bommer and Mendis (2005) that the constant (at a given response period) factors scaling 5%-damped ordinates to other damping values are inappropriate since the scaling varies with duration of shaking, and hence with magnitude and distance. Additionally, recognizing that peak ground velocity (PGV) has many applications in earthquake engineering (*e.g.*, Akkar *et al.* 2005; Akkar and Kucukdogan 2008), and that the practice of scaling PGV from 1-second spectral ordinates is highly questionable (Bommer and Alarcón 2006), we decided to simultaneously derive a PGV equation using exactly the same database and functional form (Akkar and Bommer 2007b). Although the objective of deriving the SD and PGA equations was not intended to address any shortcoming in the Ambraseys *et al.* (2005) equations, the Akkar and Bommer (2007a) equations do have three minor, but distinct, advantages: the model is effectively for pseudo-spectral acceleration rather than absolute acceleration response; the equation predicts the geometric mean of the horizontal components rather than the larger horizontal component; and the functional form includes a quadratic magnitude scaling term.

When we derived the Akkar and Bommer (2007a) equations, we followed the usual practice of plotting attenuation curves for median values of PGA and median spectral ordinates for a number of magnitude-distance scenarios, generally safely within the strict limits of applicability as defined by the range of the dataset. Additional reassurance was obtained by comparing these median values to those from other equations, including the Next Generation of Attenuation (NGA) models (Stafford, Strasser, and Bommer 2008; Bommer *et al.*, 2010). However, we have subsequently received feedback from hazard analysts and earthquake engineers who have used the equations in practice and encountered unusual features that our simplistic plots and comparisons had not revealed. More recently, serious doubts have been cast on the way the equations were derived with the development of innovative and powerful visualization tools that enable comparison of ground-motion prediction equations in terms of the full distribution of predicted values at several response periods simultaneously for ranges of magnitude and distance (Scherbaum *et al.*, forthcoming). These tools revealed that the predicted distributions from the Akkar and Bommer (2007a) equations are not at all close to those obtained from the NGA models of Abrahamson and Silva (2008), Boore and Atkinson (2008), Campbell and Bozorgnia (2008), and Chiou and Youngs (2008). This in itself might not be a particularly unsettling result (even though it would undermine our view that ground motions in active regions of shallow crustal seismicity are broadly similar), but it came as quite a surprise in light of a recent study that showed that the NGA equations provided a good fit to the European database used to derive the Akkar and Bommer (2007a, b) equations (Stafford, Strasser, and Bommer 2008). The key issue, as noted in the opening paragraph and explained in greater detail below, was the model adopted by Akkar and Bommer (2007a, b) for the aleatory variability.

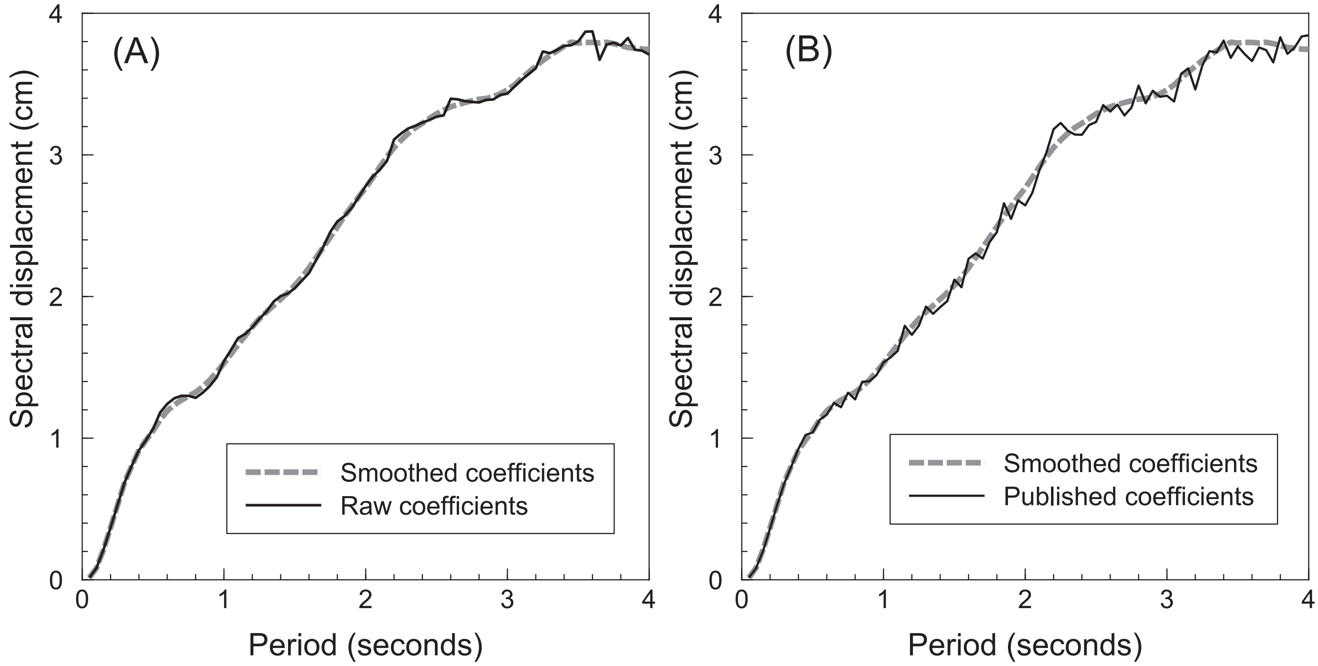
In light of these revelations, we have revisited and revised the equations for PGA, PGV, and response spectral ordinates to address the shortcomings that have been identified. The next section provides an overview of the issues with the previously published equations, after which the new equations are presented. The paper concludes with some brief notes regarding the use of the new equations, as well as recapping the lessons learned from this experience.

ISSUES WITH RECENT EUROPEAN EQUATIONS

From the outset, it should be stated that happily the problems identified are neither associated with the strong-motion records themselves nor with the metadata. The problems are related rather to the treatment of the coefficients and to the assumed model for the aleatory variability; the functional form for the model predicting median values is not called into question. We briefly explain the problems, not least because this might be valuable for others deriving empirical GMPEs.

Smoothing and Truncating Coefficients

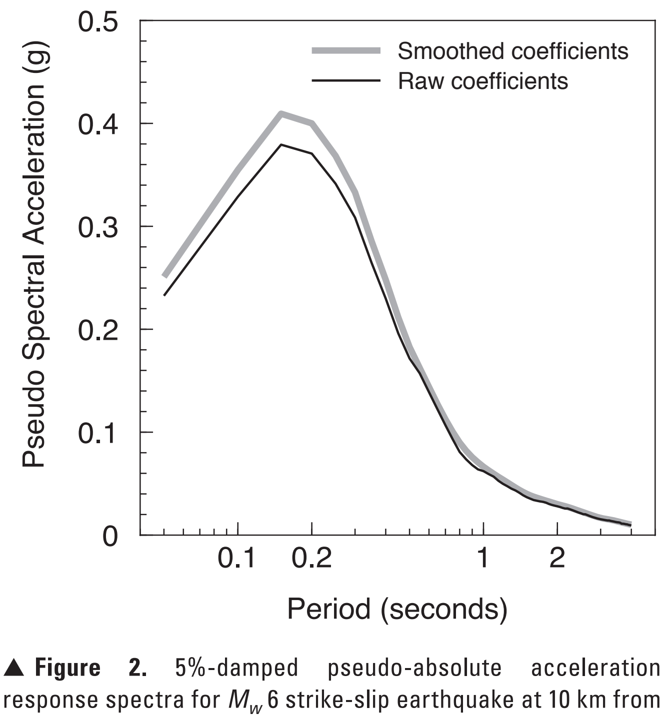
An engineering firm employing the Akkar and Bommer (2007a) equations for a site-specific hazard analysis found that the predicted displacement spectral ordinates had a rather jagged appearance in contrast to the smoothed spectra shown in figures in the published paper. The smoothing of the coefficients against period results in less jagged response spectra (Figure 1A), and the quality assurance procedures of this firm require them to reproduce the published figures for any model to be used on a project. However, the predicted spectra were found to be even less smooth than those obtained with the original regression coefficients (Figure 1B). We found that the differences were due to the fact that for the plots in our paper, we used the smoothed coefficients as we had obtained them, whereas in the paper—for reasons of space and possibly in response to comments from a reviewer or editor—we had truncated the number of decimal places to three for all of the coefficients (as opposed to four or five as derived). With the exception of the coefficient on the quadratic magnitude term, the truncation of any of the coefficients results in the perturbation of the spectral ordinates illustrated in Figure 1B.



▲ **Figure 1.** A) 5%-damped displacement response spectra for M_w 6 strike-slip earthquake at 10 km from a rock site obtained using the raw and smoothed coefficients; B) the same comparison using the smoothed coefficients and the published versions truncated to three decimal places.

This feature may have gone unnoticed previously because most users will have generated pseudo-acceleration response spectra, in which these fluctuations are less apparent. However, in exploring this issue, and looking specifically at the acceleration ordinates, we noticed that the smoothing does result, in some cases, in rather large changes to the short-period spectral amplitudes (Figure 2). Since we focused on smoothing coefficients for SD, our attention was drawn toward intermediate and long response periods, and so we missed these undesirable changes to the short-period spectral accelerations.

At this point we can draw two key conclusions regarding any new equations: the coefficients should be presented without smoothing (users can apply smoothing as appropriate for their applications), and all coefficients should be presented with five decimal places, however much we might feel this is conveying a false sense of precision.

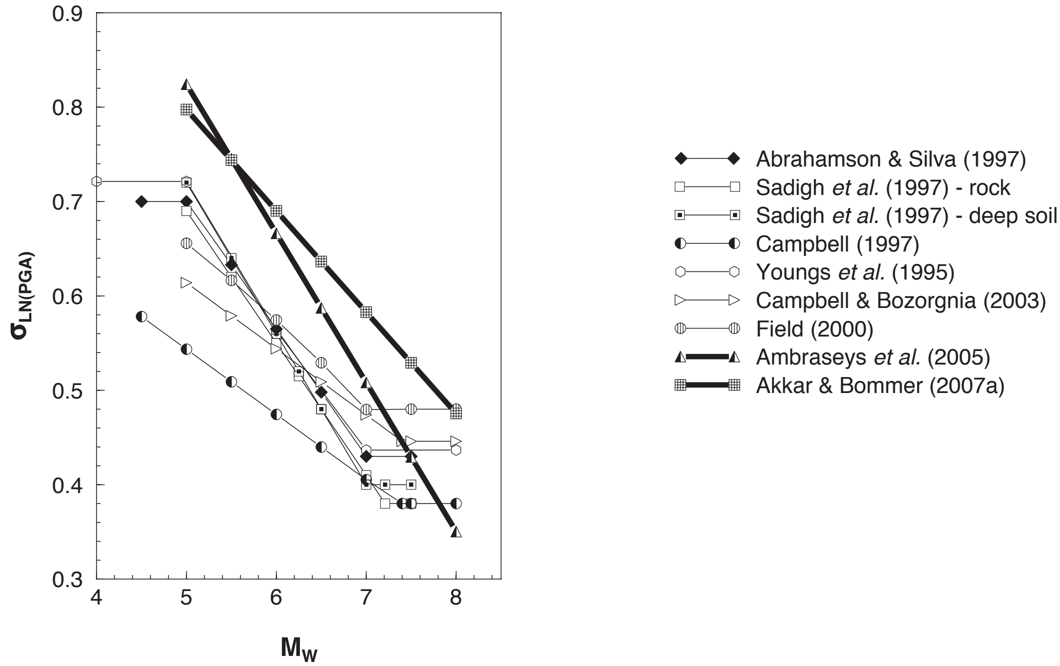


▲ **Figure 2.** 5%-damped pseudo-absolute acceleration response spectra for M_w 6 strike-slip earthquake at 10 km from a rock site obtained using the raw and smoothed coefficients.

Heteroscedastic Aleatory Variability

The issues discussed above are unlikely in themselves to have prompted the derivation and publication of modified equations; a technical note or erratum (and apology) would have sufficed. The more serious problem with the Akkar and Bommer (2007a) equations is related to the aleatory variability as characterized by the standard deviation (commonly referred to as sigma, σ). Following the identification of magnitude dependence of sigma by Youngs *et al.* (1995), many GMPEs have been derived with this feature, which is sometimes referred to as heteroscedacity (as opposed to homoscedacity in which the sigma value is constant). Both Ambraseys *et al.* (2005) and Akkar and Bommer (2007a, b) found magnitude-dependence using pure error analysis applied to the binned data (Douglas and Smit 2001), and consequently derived equations with heteroscedastic sigma using weighted regression. Figure 3 compares the magnitude-dependent sigma values of PGA equations with heteroscedastic variability, in which it can be appreciated that the European equations have somewhat higher values in general.

In addition to the values being higher, it can also be seen that all of the non-European equations model the sigma values



▲ **Figure 3.** Magnitude-dependent sigma values from several ground-motion prediction equations; the two European equations are highlighted by thicker black lines. Modified from Strasser *et al.* (2009).

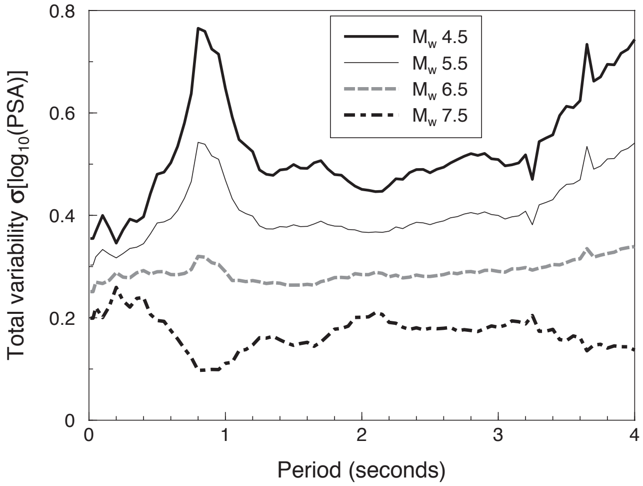
as being constant for magnitudes above a certain level; moreover, those that extend to smaller magnitudes are also adjusted to become independent of magnitude below a certain level. Similarly, the NGA equations that included magnitude-dependent sigma have constant sigma for magnitudes below 5 and above 7 (Abrahamson *et al.* 2008). In the derivation of the European models, the data was allowed to dictate the magnitude-dependence of sigma across the entire magnitude range, without any truncation or adjustment. As a result, the degree of magnitude-dependence of sigma in the Akkar and Bommer (2007a) equations varies considerably across the range of response periods (Figure 4). In Figure 4, it can be seen that at periods just below 1 second, the slope of the magnitude dependence becomes very pronounced and results in very small sigma values at M_w 7.5 and absurdly large values at M_w 4.5. Of course, this lower magnitude value is outside the strict range of applicability of the equations, but we acknowledge that such extrapolations are commonly made in the practice of seismic hazard analysis.

Subsequent investigations have cast increasing doubts on the degree of magnitude dependence of the sigma values. Bommer *et al.* (2007) showed that the degree of dependence found using the pure error approach of Douglas and Smit (2001) is highly sensitive to the size of the magnitude-distance bin in which the variability is measured. Bommer *et al.* (2007) also found, when deriving equations using a dataset extended to a lower magnitude limit of M_w 3—as opposed to M_w 5 in Akkar and Bommer (2007a)—that the heteroscedastic model yielded prohibitively high sigma values. Additionally, with the extended magnitude range dataset, even the homoscedastic model resulted in very large sigma values, leading us to suspect that much of the apparent magnitude dependence of the variability is due to limited data in the large-magnitude range and poorly determined metadata for the smaller-magnitude earthquake data. Bommer *et al.* (2007) also found that the magnitude-dependence found with many, if not most, of the magnitude-distance binning schemes was not statistically significant. In this respect, it may also be noted in passing that Ambraseys *et al.* (2005) found the magnitude dependence of sigma to be statistically significant only at periods up to 0.95 seconds, adopting a homoscedastic variability model for higher periods, which led to an abrupt change in values at this period of 0.95 seconds (Figure 5).

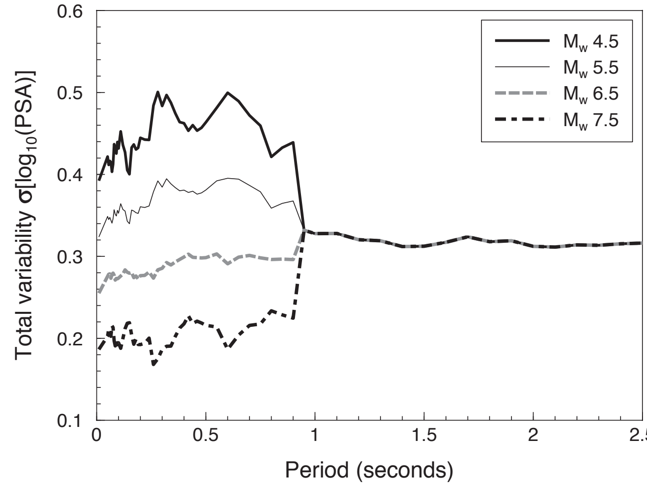
We conclude that the European data do not provide conclusive evidence of the existence of heteroscedastic variability in ground motions, and even if the magnitude-dependence is genuine, the data are insufficient to constrain this dependence reliably. One option could be to produce new equations in which the magnitude dependence of sigma is constrained so that the variability at each response period is constant at low and high magnitudes, as done, for example, for several of the models in Figure 3. However, we believe that given the characteristics of the European dataset, the more appropriate response is to derive new equations assuming homoscedastic variability, in other words with magnitude-independent sigma values.

NEW PREDICTIVE EQUATIONS

We use exactly the same dataset as used by Akkar and Bommer (2007a), which is described in some detail in Akkar and Bommer (2007b). The dataset consists of 532 accelerograms recorded at distances of up to 100 km from 131 earthquakes with magnitudes from M_w 5 to M_w 7.6. The functional form adopted is exactly the same as that used in the Akkar and Bommer (2007a, b) studies, except that we now derive equa-



▲ **Figure 4.** Total sigma values from the equations of Akkar and Bommer (2007a) at different response periods for four magnitudes.



▲ **Figure 5.** Total sigma values from the equations of Ambraseys *et al.* (2005) at different response periods for four magnitudes.

tions for the prediction of the 5%-damped pseudo-spectral acceleration, PSA, in units of cm/s^2, instead of SD:

$$\log(\text{PSA}) = b_1 + b_2 M + b_3 M^2 + (b_4 + b_5 M)\log\sqrt{R_{jb}^2 + b_6^2} + b_7 S_S + b_8 S_A + b_9 F_N + b_{10} F_R + \varepsilon\sigma, \tag{1}$$

where S_S and S_A take the value of 1 for soft (V_{s30} < 360 m/s) and stiff soil sites, otherwise zero, rock sites being defined as having V_{s30} > 750 m/s; similarly F_N and F_R take the value of unity for normal and reverse faulting earthquakes respectively, otherwise zero; ε is the number of standard deviations above or below the mean value of log(PSA). As in the original model, the one-stage maximum likelihood method of Joyner and Boore (1993) was used to compute the coefficients. The variability is decomposed into an inter-event (σ_2) and an intra-event (σ_1) component, the total standard deviation, σ, being given by the square root of the sum of their squares:

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}\,. \tag{2}$$

The values of the coefficients for median pseudo-spectral accelerations and the associated standard deviations are presented in Table 1.

The coefficients are presented without smoothing, and with five decimal places in all cases. A digital data file with these coefficients is available in the electronic supplement to this article or by request from the corresponding author.

Since the values of sigma associated with the Akkar and Bommer (2007a) equations have been the primary motivation for this new study, the first check is to examine the new sigma values, which are shown in Figure 6. The figure also compares the total sigma values with those from Ambraseys *et al.* (1996), which are comparable although slightly lower than those from the new equations; this is a little surprising, especially since being based on the larger horizontal component rather than the geometric mean component, the sigma values would be expected to be marginally higher (Beyer and Bommer 2006, 2007). The important observation is that the sigma values of our new equations are of the expected order and do not display any large fluctuations across the period range; the variation with period mimics closely that found by Ambraseys *et al.* (1996), suggesting that this is a genuine feature of the dataset. However, there is a very pronounced jump in the sigma values—most notably in the inter-event variability—at about 3.2 seconds. This corresponds to a period at which there is a sudden and dramatic reduction in the number of records used in the regression analysis as a result of the defined maximum usable period, which is some fraction of the long-period filter cut-off, determined by whether it is an analog or digital record and the site class (Akkar and Bommer 2006). Just beyond the response period of about 3.2 seconds, the number of usable records reduces by almost 100 (see Figure 2 of Akkar and Bommer 2007a), which is a significant change in the dataset underlying the equations for spectral accelerations at two closely spaced periods. This raised concerns about the coefficients for periods beyond 3 seconds, which we further discuss later in this paper.

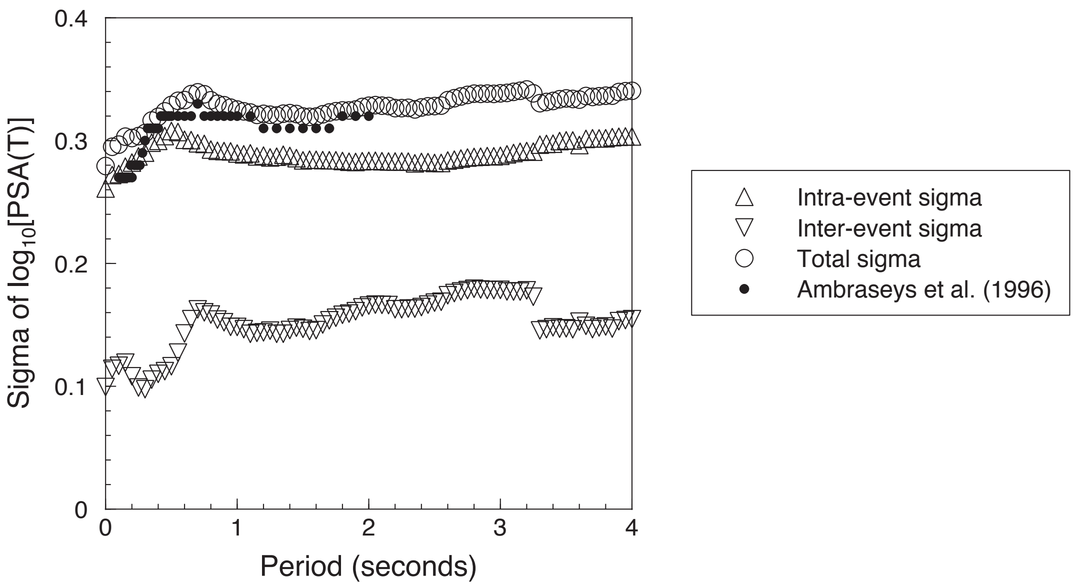
Having established that the sigma values are reasonably stable and within the expected range, the next logical step for inspecting our equations is to examine the residuals. Before continuing, we note that in mentioning expected ranges of values our judgment is being conditioned, but in this case this is not necessarily undesirable because sigma values generally fall within fairly narrow limits, as shown by Strasser *et al.* (2009). We examined total residuals against magnitude and distance, and found that no trends were apparent. We additionally explored the inter-event residuals against magnitude and the intra-event residuals against distance (Figure 7); we also looked at intra-event residuals against magnitude (not shown), which did not reveal any trends at all. The residuals are shown in Figure 7 for PGA, PGV, and spectral ordinates at 1.0 and 2.0 seconds, by way of illustration. Clearly no consistent trends can be seen that would suggest that the models are poorly conditioned, although the inter-event

TABLE 1
Coefficients of Equations 1 and 2 for Prediction of Pseudo-Spectral Accelerations

T	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	σ_1	σ_2
0.00	1.04159	0.91333	–0.08140	–2.92728	0.28120	7.86638	0.08753	0.01527	–0.04189	0.08015	0.2610	0.0994
0.05	2.11528	0.72571	–0.07351	–3.33201	0.33534	7.74734	0.04707	–0.02426	–0.04260	0.08649	0.2720	0.1142
0.10	2.11994	0.75179	–0.07448	–3.10538	0.30253	8.21405	0.02667	–0.00062	–0.04906	0.07910	0.2728	0.1167
0.15	1.64489	0.83683	–0.07544	–2.75848	0.25490	8.31786	0.02578	0.01703	–0.04184	0.07840	0.2788	0.1192
0.20	0.92065	0.96815	–0.07903	–2.49264	0.21790	8.21914	0.06557	0.02105	–0.02098	0.08438	0.2821	0.1081
0.25	0.13978	1.13068	–0.08761	–2.33824	0.20089	7.20688	0.09810	0.03919	–0.04853	0.08577	0.2871	0.0990
0.30	–0.84006	1.37439	–0.10349	–2.19123	0.18139	6.54299	0.12847	0.04340	–0.05554	0.09221	0.2902	0.0976
0.35	–1.32207	1.47055	–0.10873	–2.12993	0.17485	6.24751	0.16213	0.06695	–0.04722	0.09003	0.2983	0.1054
0.40	–1.70320	1.55930	–0.11388	–2.12718	0.17137	6.57173	0.21222	0.09201	–0.05145	0.09903	0.2998	0.1101
0.45	–1.97201	1.61645	–0.11742	–2.16619	0.17700	6.78082	0.24121	0.11675	–0.05202	0.09943	0.3037	0.1123
0.50	–2.76925	1.83268	–0.13202	–2.12969	0.16877	7.17423	0.25944	0.13562	–0.04283	0.08579	0.3078	0.1163
0.55	–3.51672	2.02523	–0.14495	–2.04211	0.15617	6.76170	0.26498	0.14446	–0.04259	0.06945	0.3070	0.1274
0.60	–3.92759	2.08471	–0.14648	–1.88144	0.13621	6.10103	0.27718	0.15156	–0.03853	0.05932	0.3007	0.1430
0.65	–4.49490	2.21154	–0.15522	–1.79031	0.12916	5.19135	0.28574	0.15239	–0.03423	0.05111	0.3004	0.1546
0.70	–4.62925	2.21764	–0.15491	–1.79800	0.13495	4.46323	0.30348	0.15652	–0.04146	0.04661	0.2978	0.1626
0.75	–4.95053	2.29142	–0.15983	–1.81321	0.13920	4.27945	0.31516	0.16333	–0.04050	0.04253	0.2973	0.1602
0.80	–5.32863	2.38389	–0.16571	–1.77273	0.13273	4.37011	0.32153	0.17366	–0.03946	0.03373	0.2927	0.1584
0.85	–5.75799	2.50635	–0.17479	–1.77068	0.13096	4.62192	0.33520	0.18480	–0.03786	0.02867	0.2917	0.1543
0.90	–5.82689	2.50287	–0.17367	–1.76295	0.13059	4.65393	0.34849	0.19061	–0.02884	0.02475	0.2915	0.1521
0.95	–5.90592	2.51405	–0.17417	–1.79854	0.13535	4.84540	0.35919	0.19411	–0.02209	0.02502	0.2912	0.1484
1.00	–6.17066	2.58558	–0.17938	–1.80717	0.13599	4.97596	0.36619	0.19519	–0.02269	0.02121	0.2895	0.1483
1.05	–6.60337	2.69584	–0.18646	–1.73843	0.12485	5.04489	0.37278	0.19461	–0.02613	0.01115	0.2888	0.1465
1.10	–6.90379	2.77044	–0.19171	–1.71109	0.12227	5.00975	0.37756	0.19423	–0.02655	0.00140	0.2896	0.1427
1.15	–6.96180	2.75857	–0.18890	–1.66588	0.11447	5.08902	0.38149	0.19402	–0.02088	0.00148	0.2871	0.1435
1.20	–6.99236	2.73427	–0.18491	–1.59120	0.10265	5.03274	0.38120	0.19309	–0.01623	0.00413	0.2878	0.1439
1.25	–6.74613	2.62375	–0.17392	–1.52886	0.09129	5.08347	0.38782	0.19392	–0.01826	0.00413	0.2863	0.1453
1.30	–6.51719	2.51869	–0.16330	–1.46527	0.08005	5.14423	0.38862	0.19273	–0.01902	–0.00369	0.2869	0.1427
1.35	–6.55821	2.52238	–0.16307	–1.48223	0.08173	5.29006	0.38677	0.19082	–0.01842	–0.00897	0.2885	0.1428
1.40	–6.61945	2.52611	–0.16274	–1.48257	0.08213	5.33490	0.38625	0.19285	–0.01607	–0.00876	0.2875	0.1458
1.45	–6.62737	2.49858	–0.15910	–1.43310	0.07577	5.19412	0.38285	0.19161	–0.01288	–0.00564	0.2857	0.1477
1.50	–6.71787	2.49486	–0.15689	–1.35301	0.06379	5.15750	0.37867	0.18812	–0.01208	–0.00215	0.2839	0.1468
1.55	–6.80776	2.50291	–0.15629	–1.31227	0.05697	5.27441	0.37267	0.18568	–0.00845	–0.00047	0.2845	0.1450
1.60	–6.83632	2.51009	–0.15676	–1.33260	0.05870	5.54539	0.36952	0.18149	–0.00533	–0.00006	0.2844	0.1457
1.65	–6.88684	2.54048	–0.15995	–1.40931	0.06860	5.93828	0.36531	0.17617	–0.00852	–0.00301	0.2841	0.1503
1.70	–6.94600	2.57151	–0.16294	–1.47676	0.07672	6.36599	0.35936	0.17301	–0.01204	–0.00744	0.2840	0.1537
1.75	–7.09166	2.62938	–0.16794	–1.54037	0.08428	6.82292	0.35284	0.16945	–0.01386	–0.01387	0.2840	0.1558
1.80	–7.22818	2.66824	–0.17057	–1.54273	0.08325	7.11603	0.34775	0.16743	–0.01402	–0.01492	0.2834	0.1582
1.85	–7.29772	2.67565	–0.17004	–1.50936	0.07663	7.31928	0.34561	0.16730	–0.01526	–0.01192	0.2828	0.1592
1.90	–7.35522	2.67749	–0.16934	–1.46988	0.07065	7.25988	0.34142	0.16325	–0.01563	–0.00703	0.2826	0.1611
1.95	–7.40716	2.68206	–0.16906	–1.43816	0.06525	7.25344	0.33720	0.16171	–0.01848	–0.00351	0.2832	0.1642
2.00	–7.50404	2.71004	–0.17130	–1.44395	0.06602	7.26059	0.33298	0.15839	–0.02258	–0.00486	0.2835	0.1657
2.05	–7.55598	2.72737	–0.17291	–1.45794	0.06774	7.40320	0.33010	0.15496	–0.02626	–0.00731	0.2836	0.1665
2.10	–7.53463	2.71709	–0.17221	–1.46662	0.06940	7.46168	0.32645	0.15337	–0.02920	–0.00871	0.2832	0.1663
2.15	–7.50811	2.71035	–0.17212	–1.49679	0.07429	7.51273	0.32439	0.15264	–0.03484	–0.01225	0.2830	0.1661
2.20	–8.09168	2.91159	–0.18920	–1.55644	0.08428	7.77062	0.31354	0.14430	–0.03985	–0.01927	0.2830	0.1627
2.25	–8.11057	2.92087	–0.19044	–1.59537	0.09052	7.87702	0.30997	0.14430	–0.04155	–0.02322	0.2830	0.1627
2.30	–8.16272	2.93325	–0.19155	–1.60461	0.09284	7.91753	0.30826	0.14412	–0.04238	–0.02626	0.2829	0.1633

TABLE 1 (continued)
Coefficients of Equations 1 and 2 for Prediction of Pseudo-Spectral Accelerations

T	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	b_9	b_{10}	σ_1	σ_2
2.35	–7.94704	2.85328	–0.18539	–1.57428	0.09077	7.61956	0.32071	0.14321	–0.04963	–0.02342	0.2815	0.1632
2.40	–7.96679	2.85363	–0.18561	–1.57833	0.09288	7.59643	0.31801	0.14301	–0.04910	–0.02570	0.2826	0.1645
2.45	–7.97878	2.84900	–0.18527	–1.57728	0.09428	7.50338	0.31401	0.14324	–0.04812	–0.02643	0.2825	0.1665
2.50	–7.88403	2.81817	–0.18320	–1.60381	0.09887	7.53947	0.31104	0.14332	–0.04710	–0.02769	0.2818	0.1681
2.55	–7.68101	2.75720	–0.17905	–1.65212	0.10680	7.61893	0.30875	0.14343	–0.04607	–0.02819	0.2818	0.1688
2.60	–7.72574	2.82043	–0.18717	–1.88782	0.14049	8.12248	0.31122	0.14255	–0.05106	–0.02966	0.2838	0.1741
2.65	–7.53288	2.74824	–0.18142	–1.89525	0.14356	7.92236	0.30935	0.14223	–0.05024	–0.02930	0.2845	0.1759
2.70	–7.41587	2.69012	–0.17632	–1.87041	0.14283	7.49999	0.30688	0.14074	–0.04887	–0.02963	0.2854	0.1772
2.75	–7.34541	2.65352	–0.17313	–1.86079	0.14340	7.26668	0.30635	0.14052	–0.04743	–0.02919	0.2862	0.1783
2.80	–7.24561	2.61028	–0.16951	–1.85612	0.14444	7.11861	0.30534	0.13923	–0.04731	–0.02751	0.2867	0.1794
2.85	–7.07107	2.56123	–0.16616	–1.90422	0.15127	7.36277	0.30508	0.13933	–0.04522	–0.02776	0.2869	0.1788
2.90	–6.99332	2.52699	–0.16303	–1.89704	0.15039	7.45038	0.30362	0.13776	–0.04203	–0.02615	0.2874	0.1784
2.95	–6.95669	2.51006	–0.16142	–1.90132	0.15081	7.60234	0.29987	0.13584	–0.03863	–0.02487	0.2872	0.1783
3.00	–6.92924	2.45899	–0.15513	–1.76801	0.13314	7.21950	0.29772	0.13198	–0.03855	–0.02469	0.2876	0.1785

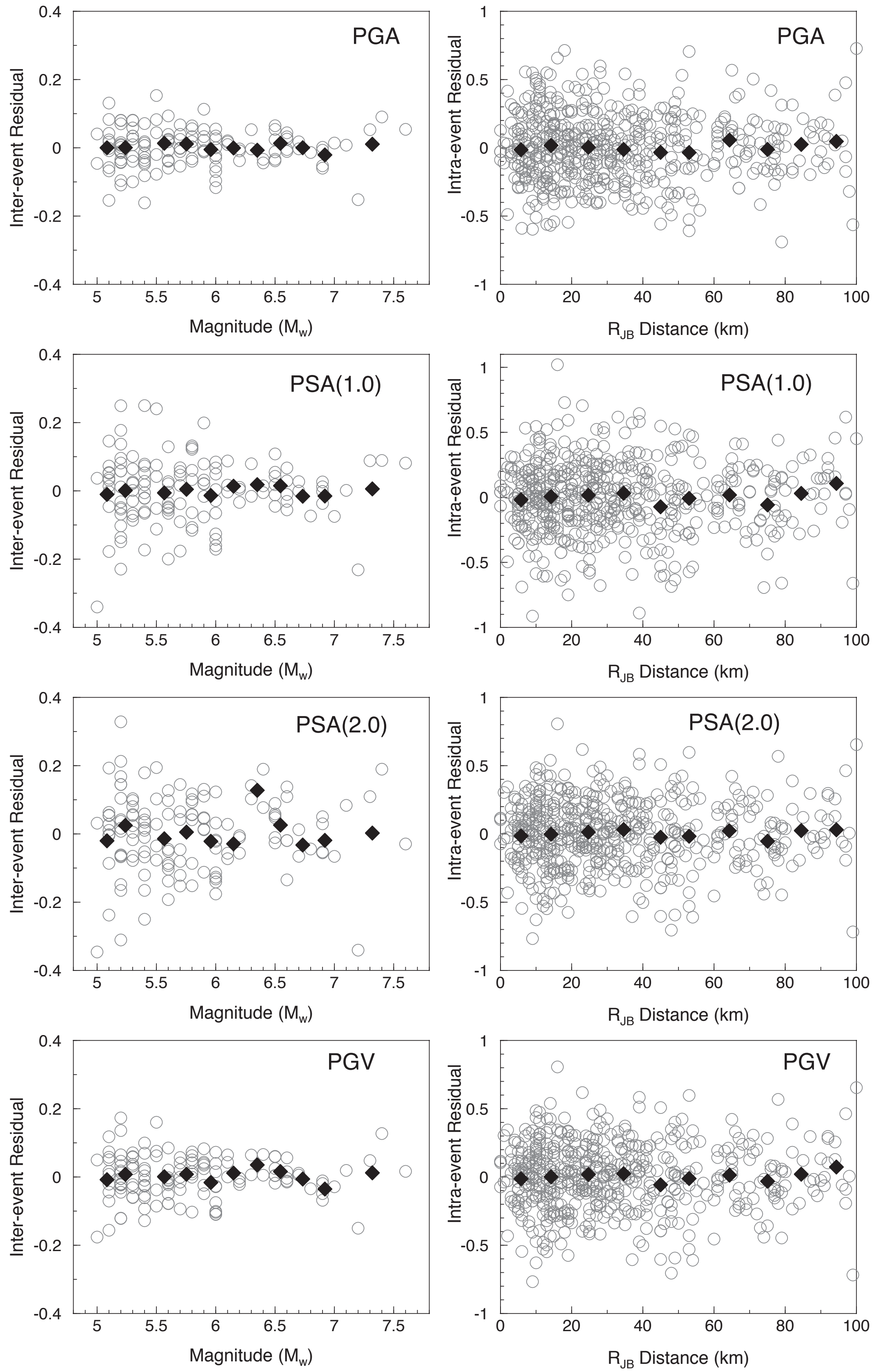


▲ **Figure 6.** Inter-event, intra-event, and total sigma values from new equations at different response periods. The total sigmas from the equations of Ambraseys *et al.* (1996) are shown for comparison.

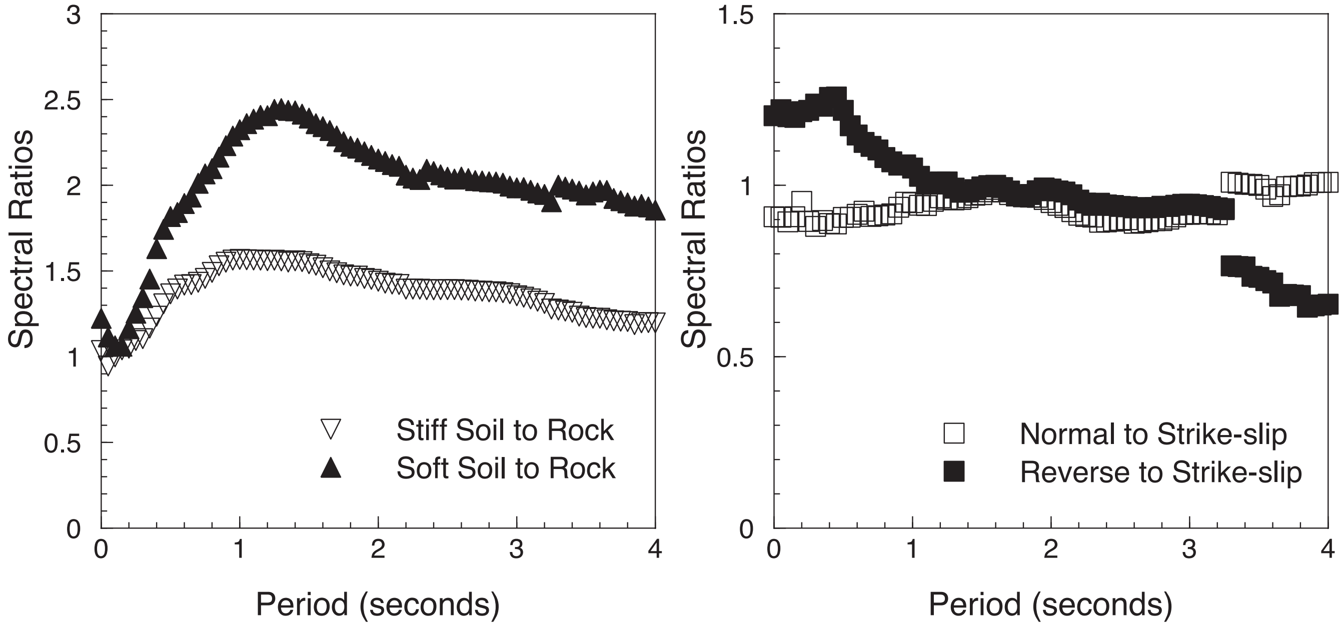
residuals do seem to become a little more erratic with increasing response period. However, these are fluctuations, which might to some extent be the result of the arbitrarily chosen magnitude bins, rather than consistent trends. Looking at the inter-event residuals for PGA and 1-second pseudo-spectral acceleration one could easily infer that the variability is magnitude-dependent, but this apparent reduction in the variability of the residuals at higher magnitudes needs to be balanced with the consideration that the data become sparse for magnitudes above 6.5. Overall, these residual plots lead us to conclude that the equations are robust and reliable, or at least this is so to the extent that the underlying metadata are well known.

To further explore the new equations, we also look at the physical implications of the coefficients. Figure 8 shows the implied amplifying effects of stiff and soft soil sites with respect to rock sites and the influence of normal and reverse fault ruptures with respect to strike-slip mechanisms. The results for site response effects look perfectly reasonable, although it must be noted that the model does not consider nonlinear soil response, not because we do not believe that it is a real phenomenon but simply because the European dataset does not reveal its presence (Akkar and Bommer 2007b).

The influence of style-of-faulting is broadly consistent with general trends identified in previous studies (*e.g.*, Bommer *et al.* 2003), but here again we see the pronounced effect of the sharp reduction in numbers of usable records at about a period of 3.2 seconds manifesting as a jump in the coefficients at this period. Although less pronounced, it is also visible in the coef-



▲ **Figure 7.** Inter- and intra-event residuals from the new equations plotted against magnitude and distance respectively, for PGA, pseudo-spectral accelerations at 1.0 and 2.0 seconds, and PGV. The black diamonds show average residuals in bins of 0.5 magnitude units and 10 km distance.

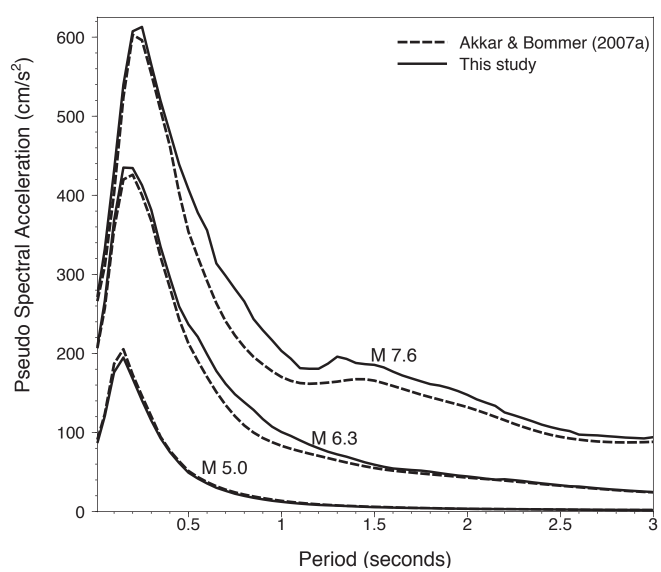


▲ **Figure 8.** Illustration of the effect of the coefficients S_A and S_S, and F_N and F_R, respectively, on predicted spectral ordinates, shown by the ratios of spectral ordinates with respect to those on rock sites (left) and with respect to strike-slip ruptures (right).

ficients for soft-soil site simplification. In view of these observations, and those noted in Figure 6 for the variability, we conclude that the equations should not be used up to 4.0 seconds since there is a very marked discontinuity at 3.2 seconds. For this reason, Table 1 only presents coefficients for periods up to 3.0 seconds.

Notwithstanding that such simple visual comparisons do not reveal the complete picture, Figure 9 shows median spectral ordinates from Akkar and Bommer (2007a) and from the new equations, for rock sites at 10 km from strike-slip ruptures of three different magnitudes; the latter values are chosen to represent the limits, and the center, of our dataset. One needs to be a little cautious in making such a comparison, because it is important to know what the expectations are and whether these expectations are well founded. On the one hand, the equations use the same dataset, functional form, and regression technique, which means that we would expect the predicted median spectra to be similar. On the other hand, the assumptions about the variability model influence the coefficients; therefore we would not expect them to be identical. The predicted medians are very similar, the differences increasing with earthquake magnitude. Figure 10 makes exactly the same comparison except that instead of plotting median pseudo-spectral accelerations we present 84th-percentile values, something that is not done very often but which is possibly more informative than plots like those in Figure 9.

In this case, the results show that the spectra at M_w 5.0 and M_w 6.3 are very similar from the previous and revised equations, but quite dramatically different at M_w 7.6. The new equations certainly predict spectral ordinates whose trends are more consistent and which conform better to our expectations. At a period of about 0.8 seconds, the Akkar and Bommer (2007a) equations predict the same median-plus-one-



▲ **Figure 9.** Comparison of median pseudo-spectral accelerations predicted for rock sites at 10 km from the source of strike-slip earthquakes of different magnitudes obtained from the equations of Akkar and Bommer (2007a) and the new equations presented herein.

standard-deviation level of pseudo-spectral acceleration for M_w 6.3 and M_w 7.6 earthquakes at the same distance and for the same site conditions. This result, which is somewhat counterintuitive, is probably due to the excessively small sigma value for the larger magnitude earthquake as a result of the very pronounced magnitude-dependence modeled at this period.

The equation for peak ground velocity, in cm/s, has exactly the same functional form, with the following coefficients for median values:

$$\log(PGV) = -2.12833 + 1.21448M - 0.08137M^2 \\ -(2.46942 - 0.22349M)\log\sqrt{R_{jb}^2 + 6.41443^2} \\ +0.20354S_S + 0.08484S_A - 0.05856F_N \\ +0.01305F_R \quad (3)$$

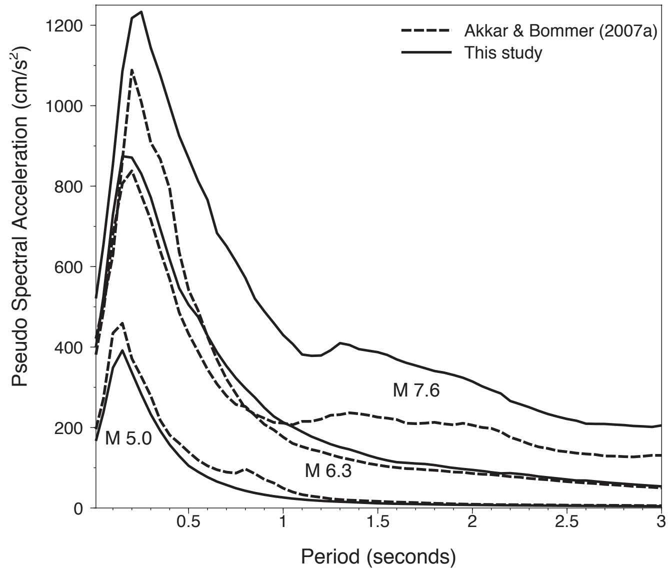
The associated standard deviations are σ_1 = 0.2562 and σ_2 = 0.1083, whereas the total standard deviation is 0.278. In the heteroscedastic model of Akkar and Bommer (2007b), the total sigma value varies from 0.387 at M_w 5.0 to 0.121 at M_w 7.6. Figure 11 shows predicted median values of PGV against distance for earthquakes at the upper and lower magnitude limits of the dataset, to illustrate the influence of the site-effects terms and the style-of-faulting.

Once again, the trends are as expected, although for this parameter reverse faults are expected to produce motions only fractionally higher than those from strike-slip earthquakes. The equation of Akkar and Bommer (2007b) has the same characteristics.

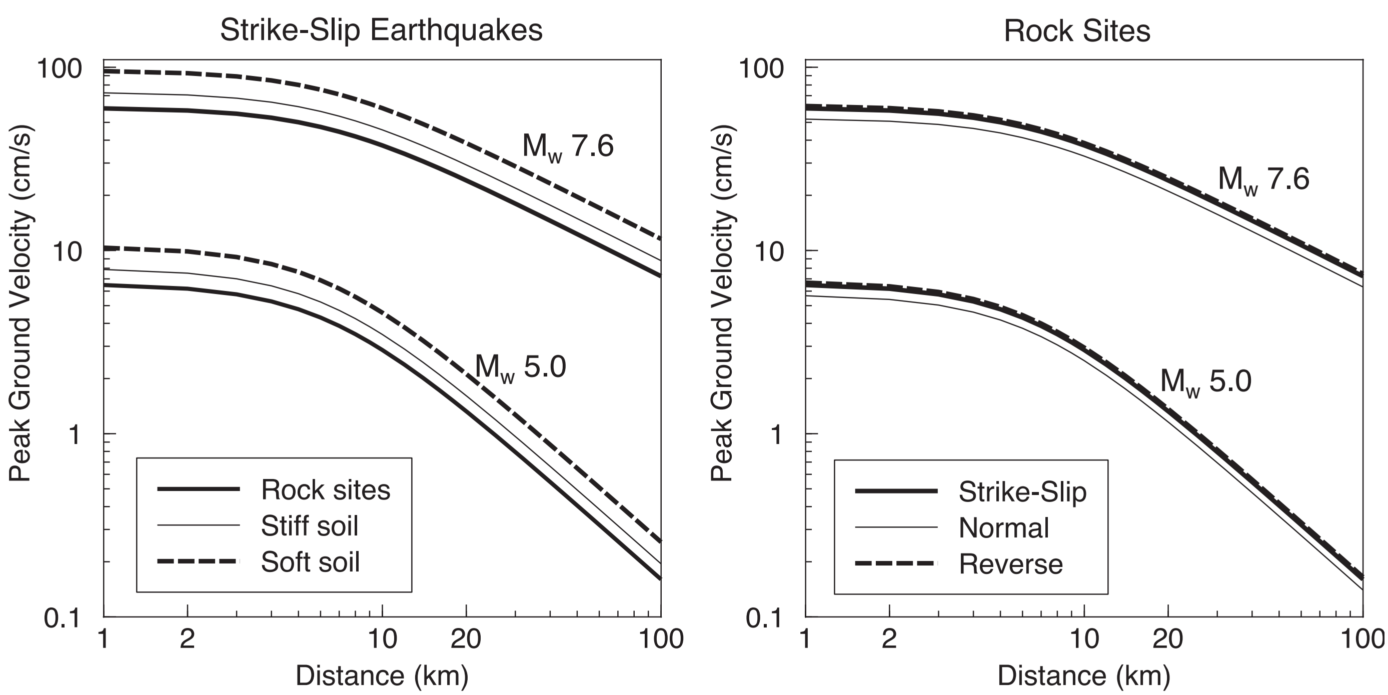
DISCUSSION AND CONCLUSIONS

Our first conclusion must be to reiterate the warnings to others from our own experience and mistakes, assuming that others could as easily walk into the same problems. The first, and more minor, warning regards the truncation of coefficients and the urge to express all numbers to no more than three decimal places, which can have a surprisingly large impact on the appearance of the resulting response spectra. The second, and more serious, warning regards assessing empirical equations only by plotting median motions and in particular for those magnitude-distance ranges well within the “comfort zone” of the equation as determined by the distribution of the dataset.

The coefficients for spectral ordinates at response periods from 0 to 3 seconds have been presented herein without



▲ **Figure 10.** Comparison of 84th-percentile pseudo-spectral accelerations predicted for rock sites at 10 km from the source of strike-slip earthquakes of different magnitudes obtained from the equations of Akkar and Bommer (2007a) and the new equations presented herein.



▲ **Figure 11.** Predicted median values of peak ground velocity for different combinations of magnitude, style-of-faulting, distance and site classification. The two magnitudes are the limiting values of the underlying dataset.

smoothing, and users may wish to apply a smoothing function before the equations are deployed.

We propose that the new equations presented in this paper be used instead of those presented previously by Akkar and Bommer (2007a, b). One aspect that is thereby lost is the direct prediction of spectral ordinates for damping values other than 5% of critical. However, this can be easily remedied. If one wishes to account for the variation in the scaling of the 5%-damped ordinates to other target damping levels in terms of magnitude and distance, use can be made of the relationships derived by Cameron and Green (2007) or alternatively the variation could be inferred from the ratios of median values predicted by the equations of Akkar and Bommer (2007a). Alternatively, the variation of the scaling factors can be directly modeled as a function of duration or number of cycles (Stafford, Mendis, and Bommer 2008). The duration of motion for different earthquake scenarios can be calculated using the empirical equations of Bommer *et al.* (2009) and the numbers of cycles of motion from the equations of Stafford and Bommer (2009).

Another limitation of the new equations is that they are recommended for use only up to a period of 3 seconds, whereas the previous equations extended to 4 seconds. However, in those applications where the response at long periods is of interest, use can be made of the NGA equations (which extend to 10 seconds), especially since these have been shown to be applicable in the European, Mediterranean, and Middle Eastern regions (Stafford, Strasser, and Bommer 2008). In any case, epistemic uncertainty in the median ground motion means that hazard analyses should never be conducted using a single GMPE but rather a number of these equations should be combined within a logic-tree framework (Bommer *et al.* 2005). For hazard studies in Europe, we would recommend the use of these new equations in combination with one or more of the NGA models; since both the NGA and the new European models use the same parameter definitions in most cases, issues of compatibility are largely resolved. Whether or not additional equations derived from local data are also included in the logic tree must be the choice of the hazard analyst.

A final point is that nominally the range of applicability of these new equations is for distances up to 100 km and for earthquakes of magnitudes between 5.0 and 7.6; it is inevitable, in probabilistic seismic hazards analysis (PSHA), that the equations will be extrapolated beyond these limits, but the user should be aware of this and, if necessary, adjust the branches of the logic tree to capture the greater epistemic uncertainty associated with predictions for events beyond the bounds of the dataset. However, the user should also be aware that the strict range of applicability of the equations may actually be smaller than the magnitude range of the dataset, since it has been found that empirical GMPEs tend to overpredict ground motions for earthquakes at the lower magnitude limit of the data. This has been shown recently for European equations (Bommer *et al.* 2007) and for the NGA models for California (*e.g.*, Atkinson and Morrison 2009). The next stage of our work will be to extend the European model to smaller magnitudes, possibly following the approach applied by Chiou *et al.* (forthcoming) to the NGA model of Chiou and Youngs (2008). One of the aspects to be explored for these pan-European equations is the inclusion of focal depth, since we have used Joyner-Boore distance, which is measured horizontally on the surface, and we do not include a depth-to-top-of-rupture term as included in the NGA models. For small-magnitude events, with rupture dimensions that are small in comparison to the thickness of the seismogenic crust, the depth to the rupture could be expected to exert a strong influence on the amplitude of ground motions.

The first stage of this work will be to derive pan-European equations for a wide range of magnitudes, noting that the equations of Bommer *et al.* (2007) derived for M_w 3.0 to M_w 7.6 only covered response periods up to 0.5 seconds and were produced as part of an exploratory exercise rather than for practical application. Should regional differences in the motions from small-magnitude earthquakes in different parts of southern Europe, the Mediterranean, and Middle East be clearly identified, then subsequent extension of the work could be to adjust the pan-European model to be applicable to specific regions at lower magnitudes.

ACKNOWLEDGMENTS

We are indebted to various practitioners for providing us with useful feedback on the performance of the 2007 equations when employed in seismic hazard analyses. The first person to raise the issue of the period-to-period fluctuations of sigma was Malcolm Goodwin of ABS Consulting, and the effect of truncating the decimal places of the coefficients was identified by Dr. Antonio Fernández and colleagues at Paul C. Rizzo Associates. The most convincing evidence of the need to revisit and update the equations was provided by Professor Frank Scherbaum and his colleague Nicolas Kuehn, from the University of Potsdam, who included the equations in self-organizing maps together with the NGA equations and showed us that we were not where we expected to be on the map! The original manuscript was considerably improved by insightful and constructive review comments from Hilmar Bungum and John Douglas, and we also benefited from useful suggestions given by Bob Youngs and Emrah Yenier. In addition to expressing our thanks, it would be remiss of us if we did not also offer an apology to all those who have used the previously published equations and may now need to revise or reconsider their results.

REFERENCES

Abrahamson, N. A., and W. Silva (1997). Empirical response spectral attenuation relations for shallow crustal earthquakes. *Seismological Research Letters* **68** (1), 94–127.

Abrahamson, N., G. Atkinson, D. Boore, Y. Bozorgnia, K. Campbell, B. Chiou, I. M. Idriss, W. Silva, and R. Youngs (2008). Comparisons of the NGA ground-motion relations. *Earthquake Spectra* **24** (1), 45–66.

Abrahamson, N. A., and W. Silva (2008). Summary of the Abrahamson & Silva NGA ground-motion relations. *Earthquake Spectra* **24** (1), 67–97.

Akkar, S., and J. J. Bommer (2006). Influence of long-period filter cut-off on elastic spectral displacements. *Earthquake Engineering & Structural Dynamics* **35** (9), 1,145–1,165.

Akkar, S., and J. J. Bommer (2007a). Prediction of elastic displacement response spectra in Europe and the Middle East. *Earthquake Engineering & Structural Dynamics* **36** (10), 1,275–1,301.

Akkar, S., and J. J. Bommer (2007b). Empirical prediction equations for peak ground velocity derived from strong-motion records from Europe and the Middle East. *Bulletin of the Seismological Society of America* **97** (2), 511–530.

Akkar, S., and B. Kucukdogan (2008). Direct use of PGV for estimating peak nonlinear oscillator displacements. *Earthquake Engineering & Structural Dynamics* **37** (12), 1,411–1,433.

Akkar, S., H. Sucuoglu, and A. Yakut (2005). Displacement-based fragility functions for low- and mid-rise ordinary concrete buildings. *Earthquake Spectra* **21** (4), 901–927.

Ambraseys, N. N., J. Douglas, S. K. Sarma, and P. Smit (2005). Equations for the estimation of strong ground motions from shallow crustal earthquakes using data from Europe and the Middle East: Horizontal peak ground acceleration and spectral acceleration. *Bulletin of Earthquake Engineering* **3** (1), 1–53.

Ambraseys, N. N., K. A. Simpson, and J. J. Bommer (1996). Prediction of horizontal response spectra in Europe. *Earthquake Engineering & Structural Dynamics* **25** (4), 371–400.

Ambraseys, N. N, P. Smit, J. Douglas, B. Margaris, R. Sigbjörnsson, R. S. Ólafsson, P. Suhadolc, and G. Costa (2004). Internet site for European strong-motion data. *Bollettino di Geofisica Teorica ed Applicata* **45** (3), 113–129. In English.

Atkinson, G. M., and M. Morrison (2009). Observations on regional variability in ground-motion amplitudes for small-to-moderate earthquakes in North America. *Bulletin of the Seismological Society of America* **99** (4), 2,393–2,409.

Beyer, K., and J. J. Bommer (2006). Relationships between median values and between aleatory variabilities for different definitions of the horizontal component of motion. *Bulletin of the Seismological Society of America* **96** (4A), 1,512–1,522.

Beyer, K., and J. J. Bommer (2007). Erratum: Relationships between median values and between aleatory variabilities for different definitions of the horizontal component of motion. *Bulletin of the Seismological Society of America* **97** (5), 1,769.

Bommer, J. J., and J. E. Alarcón (2006). The prediction and use of peak ground velocity. *Journal of Earthquake Engineering* **10** (1), 1–31.

Bommer, J. J., J. Douglas, and F. O. Strasser (2003). Style-of-faulting in ground-motion prediction equations. *Bulletin of Earthquake Engineering* **1** (2), 171–203.

Bommer, J. J., and R. Mendis (2005). Scaling of spectral displacement ordinates with damping ratios. *Earthquake Engineering & Structural Dynamics* **34** (2), 145–165.

Bommer, J. J., F. Scherbaum, H. Bungum, F. Cotton, F. Sabetta, and N. A. Abrahamson (2005). On the use of logic trees for ground-motion prediction equations in seismic hazard analysis. *Bulletin of the Seismological Society of America* **95** (2), 377–389.

Bommer, J. J., P. J. Stafford, and S. Akkar (2010). Current empirical ground-motion prediction equations for Europe and their application to Eurocode 8. *Bulletin of Earthquake Engineering* **8** (1), 5–26.

Bommer, J. J., P. J. Stafford, J. E. Alarcón, and S. Akkar (2007). The influence of magnitude range on empirical ground-motion prediction. *Bulletin of the Seismological Society of America* **97** (6), 2,152–2,170.

Bommer, J. J., P. J. Stafford, and J. E. Alarcón (2009). Empirical equations for the prediction of the significant, bracketed and uniform duration of earthquake ground motion. *Bulletin of the Seismological Society of America* **99** (6), 3,217–3,233.

Boore, D. M., and G. M. Atkinson (2008). Ground-motion prediction equations for the average horizontal component of PGA, PGV, and 5%-damped PSA at spectral periods between 0.1 s and 10.0 s. *Earthquake Spectra* **24** (1), 99–138.

Cameron, W. I., and R. U. Green (2007). Damping correction factors for horizontal ground-motion response spectra. *Bulletin of the Seismological Society of America* **97** (3), 934–960.

Campbell, K. W. (1997). Empirical near-source attenuation of horizontal and vertical components of peak ground acceleration, peak ground velocity, and pseudo-absolute acceleration response spectra. *Seismological Research Letters* **68** (1), 154–179.

Campbell, K. W., and Y. Bozorgnia (2003). Updated near-source ground motion (attenuation) relations for the horizontal and vertical components of peak ground acceleration and acceleration response spectra. *Bulletin of the Seismological Society of America* **93** (1), 314–331.

Campbell, K. W., and Y. Bozorgnia (2008). NGA ground motion model for the geometric mean horizontal component of PGA, PGV, PGD and 5%-damped linear elastic response spectra for periods ranging from 0.1 s to 10.0 s. *Earthquake Spectra* **24** (1), 139–171.

Chiou, B. S.-J., and R. R. Youngs (2008). An NGA model for the average horizontal component of peak ground motion and response spectra. *Earthquake Spectra* **24** (1), 173–215.

Chiou, B., R. Youngs, N. Abrahamson, and K. Addo (forthcoming). Ground-motion attenuation model for small-to-moderate shallow crustal earthquakes in California and its implications on regionalization of ground-motion prediction models. *Earthquake Spectra*.

Douglas, J. (2007). On the regional dependence of earthquake response spectra. *ISET Journal of Earthquake Technology* **44** (1), 71–99.

Douglas, J., and P. M. Smit (2001). How accurate can strong ground motion attenuation relations be? *Bulletin of the Seismological Society of America* **91**, 1,917–1,923.

Field, E. H. (2000). A modified ground-motion attenuation relationship for southern California that accounts for detailed site classification and a basin-depth effect. *Bulletin of the Seismological Society of America* **90** (6B), S209–S221.

Joyner, W. B., and D. M. Boore (1993). Methods for regression analysis of strong-motion data. *Bulletin of the Seismological Society of America* **83** (2), 469–487.

Sadigh, K., C.-Y. Chang, J. A. Egan, F. Makdisi, and R. R. Youngs (1997). Attenuation relationships for shallow crustal earthquakes based on California strong motion data. *Seismological Research Letters* **68** (1), 180–189.

Scherbaum, F., N. M. Kuehn, M. Ohrnberger, and A. Koehler (forthcoming). Exploring the proximity of ground-motion models using high-dimensional visualization techniques. *Earthquake Spectra*.

Stafford, P. J., and J. J. Bommer (2009). Empirical equations for the prediction of the equivalent number of cycles of earthquake ground motion. *Soil Dynamics & Earthquake Engineering* **29** (11/12), 1,425–1,436.

Stafford, P. J., R. Mendis, and J. J. Bommer (2008). Dependence of damping correction factors for response spectra. *ASCE Journal of Structural Engineering* **134** (8), 1,364–1,373.

Stafford, P. J., F. O. Strasser, and J. J. Bommer (2008). An evaluation of the applicability of the NGA models to ground-motion prediction in the Euro-Mediterranean region. *Bulletin of Earthquake Engineering* **6** (2), 149–177.

Strasser, F. O., N. A. Abrahamson, and J. J. Bommer (2009). Sigma: Issues, insights, and challenges. *Seismological Research Letters* **80** (1), 40–56.

Youngs, R. R., N. Abrahamson, F. I. Makdisi, and K. Singh (1995). Magnitude-dependent variance of peak ground acceleration. *Bulletin of the Seismological Society of America* **85** (4), 1,161–1,176.

Imperial College London
Civil and Enviromental Engineering Department
South Kensington Campus
London, SW7 2AZ U.K.
j.bommer@imperial.ac.uk
(J. J. B.)