Repeated Moral Hazard with a Time-Inconsistent Agent*

Murat Yılmaz†

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Abstract

I consider a repeated principal-agent model with moral hazard, in which the agent has $\beta\delta$-preferences, which are widely used to capture time-inconsistency. I analyze the case where the agent is *sophisticated* in the sense that he is fully aware of his inconsistent discounting. I characterize the optimal wage scheme for such an agent and compare it to time-consistent benchmarks. The marginal cost of rewarding the agent for high output today exceeds the marginal benefit of delaying these rewards until tomorrow. In this sense, the principal does not smooth the agent's rewards over time. When facing a sophisticated agent, it is optimal for the principal to reward the good performance more and punish the bad performance more in the early period, relative to the optimal wage scheme for a time-consistent agent.

Keywords : repeated moral hazard; time-inconsistency; $\beta\delta$-preferences; sophisticated agent; naive agent.

JEL **Classification Numbers:** D03, D82, D86.

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†Boğaziçi University, Department of Economics, Bebek, Istanbul, 34342. Phone: +90 212 3597656. Fax: +90 212 2872453. Email: *muraty@boun.edu.tr.* Web site: *http://www.econ.boun.edu.tr/yilmaz*

1 Introduction

There is a considerable amount of evidence showing that agents' behavior often exhibits time-inconsistency. When the benefits of an action are in the future and the costs are immediate, agents do not give the benefits much weight. That is, they tend to postpone costly actions and tough projects (e.g. finishing up writing a paper, filing taxes or going to the gym), but rarely tend to postpone gratification.[1] Although a substantial amount of work has been done regarding individual decision making under time-inconsistency, there is little work done on economic interactions that include time-inconsistent individuals. In particular, a natural question is how a dynamically consistent principal would contract with an agent who has time-inconsistent preferences.

The focus of the current paper is on the repeated principal-agent problem where there is moral hazard, with the standard trade-off between incentives and insurance.[2] However, I depart from the standard repeated moral hazard literature in allowing the agent's preferences to be time-inconsistent, specifically present-biased. The contribution of this paper is to characterize the effects of time-inconsistency on the optimal contract and on the principal's expected profits. I show that if the agent is aware of his time-inconsistency, the principal does not smooth the agent's rewards over time. It is optimal for the principal to provide the agent with more incentives in the earlier period and less in the later period, relative to the optimal wage scheme for a time-consistent agent.

The agent's time-inconsistency is modeled by $\beta\delta$-preferences.[3] An agent with $\beta\delta$-preferences has a discount factor $\beta\delta$ between the current and the next period, and δ between any other pair of successive periods. In other words, the agent's preference for a payoff in date t over a payoff in date $t+1$ is stronger as date t gets closer. A time-inconsistent agent may or may not be aware of his inconsistency: A sophisticated agent is fully aware of his inconsistency in the sense that he correctly predicts how his future selves will behave. A naive agent, on the other hand, is not aware of his inconsistency, in the sense that he mispredicts the behavior of his future selves through an overestimated β. The focus of this paper is on the sophisticated agent. I consider the case where the agent is naive in Yılmaz (2013) where I show that the principal achieves the same expected

[1] See Frederick, Loewenstein, O'Donoghue (2002), for an extensive overview of the literature, and see Loewenstein and Prelec (1992) for an extensive survey on anomalies in intertemporal choice.

[2] See, for instance, Rogerson (1985) and Lambert (1983).

[3] $\beta\delta$-preferences are first developed by Phelps and Pollak (1968) and later used by many including Laibson (1997), O'Donoghue and Rabin (1999a,1999b,2001) among others. Also see O'Donoghue and Rabin (2000) for various economics applications based on time-inconsistency captured with $\beta\delta$-preferences.

profit from a time-inconsistent agent who is fully aware of his inconsistency and a time-inconsistent agent who is unaware of his inconsistency. In the current paper, I briefly discuss this case in Section 4 as an extension.

The principal is not able to observe the effort levels picked by the risk-averse agent; thus, she faces the natural trade-off between smoothing risk and providing incentives. In the contracting stage, period 0, the principal offers a wage scheme to the agent ensuring that he accepts it. This wage scheme also ensures that the agent exerts high effort in both periods, 1 and 2, following the contracting stage. The wage scheme exhibits memory in the sense that the wages paid in period 2 depend on the performance in period 1.

When the agent is sophisticated, the principal does not intertemporally smooth the agent's rewards. More precisely, the marginal cost of rewarding the agent today exceeds the marginal benefit of delaying these rewards until tomorrow for a high level of output and vice versa for a low level of output. This is because the agent's discount factor is changing over time. In contrast, the optimal contract for a time-consistent agent smoothes wages in this sense. Moreover, in the earlier period a sophisticated time-inconsistent agent receives bigger rewards for good performance and bigger punishments for bad performance under the optimal wage scheme, relative to a time-consistent agent. The intuition for this result is as follows. Suppose that the principal has two options for the sophisticated agent: She can either increase the incentives in period 1 or increase them in period 2, relative to the optimal contract for the time-consistent agent. To achieve the same overall incentives, the increase in period 2 must be bigger, because period 2 is discounted more by the agent. But the agent's individual rationality holds with slack under the former increase if it binds under the latter increase. This is because the agent discounts both changes to the contracting stage at the same rate since there is no payment made in the contracting stage. Thus, the principal can increase the wage differential in period 1, relative to the optimal scheme for the time-consistent agent, and still ensure that the agent accepts the contract.

When comparing the expected profit from a sophisticated agent to the one from a time-consistent agent, since the sophisticated agent discounts at either $\beta\delta$ or δ, I consider two benchmark time-consistent agents, one with a discount factor δ and one with a discount factor $\beta\delta$. I compare the expected profit from a sophisticated agent to that from these two time-consistent agents. The principal is better off with a time-consistent agent with a discount factor δ than with the sophisticated time-inconsistent agent, who in turn is better than a time-consistent agent with a discount factor $\beta\delta$, under a mild condition. This makes sense because the sophisticated agent with $\beta\delta$-preferences has an overall discount factor between δ and $\beta\delta$.[4]

This paper is related to a number of other papers in the literature in two dimensions, one regarding dynamic contracts with repeated principal-agent relationships, and the other regarding the time-inconsistent preferences. Among the papers within the dynamic contract theory, the current paper is closest to Rogerson (1985) where he considers a repeated moral hazard problem with a time-consistent agent, and shows that the optimal contract exhibits memory and that the marginal cost of rewarding the agent today equals the marginal benefit of delaying these rewards to tomorrow. The current paper challenges the main result shown in Rogerson (1985) by considering time-inconsistent preferences by the agent, and also discusses the other results and extensions. A very parallel paper to Rogerson (1985) is Lambert (1983) where he uses a first-order approach to show similar results. Spear and Srivastava (1987) also consider an infinitely repeated agency model and show that the optimal contract can be characterized by reducing it to a simple two-period constrained optimization problem. They include the agent's continuation expected utility as a state variable to show that the history-dependence can be represented in a simpler way. Another important paper on repeated principal-agent relationships is Fudenberg, Holmstorm and Milgrom (1990) where they study an infinitely repeated principal-agent relationship and show that, under some conditions, the efficient long-term contract can be implemented by a sequence of short-term contracts, hence commitments are not necessary. These conditions include no adverse selection at the time of renegotiation, payments being based on the joint information with no delay, and the agent's preferences being additively separable.[5] Among the papers that consider time-inconsistent preferences in a related context are O'Donoghue and Rabin (1999b) and Gilpatric (2008). Both consider principal-agent relationship with time-inconsistent agents. The former introduces a moral hazard problem in the form of unobservable task-cost realizations and assumes that the agent is risk-neutral. The latter focuses on a contracting problem with time-inconsistent agents assuming that profit is fully determined by the effort, so effort is effectively observable. The current paper distinguishes itself from these papers by allowing the trade-off between risk and incentives. There is also a series of papers by O'Donoghue and Rabin (1999a, 2000, 2001) in which they consider $\beta\delta$-preferences and focus on individual decision making rather than contractual relations. In DellaVigna and Malmendier (2004)

[4]In the extensions section, I also consider an alternative time-consistent agent with an average discount factor equal to the average discount factor of the sophisticated agent.

[5]Other related papers regarding dynamic contract theory are Harris and Holmstorm (1982) and Malcomson and Spinnewyn (1988).

a monopolistic firm is facing a problem of designing an optimal two-part tariff where the consumer is time-inconsistent, captured with $\beta\delta$-preferences, and the principal knows whether the agent is aware of his inconsistency or not. Concerning $\beta\delta$-preferences, Chade, Prokopovych and Smith (2008) study infinitely repeated games where players have such preferences. They characterize the equilibrium payoffs and show that the equilibrium payoff set is not monotonic in β or δ. An alternative approach to repeated principal-agent relationships involving dynamic inconsistency is provided by Eliaz and Spiegler (2006). They characterize the optimal menu of contracts when a monopoly is contracting with dynamically inconsistent agents and show that it includes exploitative contracts for naive agents. The current paper distinguishes itself from this paper through its modeling aspects.[6]

Section 2 describes the model. Section 3 provides the properties of the optimal contract for sophisticated agents and compares it to the contract for a time-consistent agent. Section 4 provides a number of extensions including continuous effort choice, the case with more than three periods, access to the credit market, an alternative benchmark time-consistent agent, relaxing the full-commitment assumption and the naive agent case. It also discusses some open questions. Some of the proofs and detailed analysis of the extensions are in Appendix A and Appendix B.

2 The Model

I consider a finitely repeated moral hazard problem. A principal is contracting with an agent to work on a two period project. Each period the agent can exert costly effort. The principal cannot observe the effort choices of the agent. The project, in each period, has an output which is publicly observed. The output in each period is stochastic and affected by the effort level picked by the agent in that period.

2.1 Timing

Unlike the standard two-period repeated moral hazard problem, there is need for at least three periods for the agent's time-inconsistent preferences to play a role. Consider the following timing of events:

Figure 1

[6] There are other related papers that study time-inconsistency in other contexts. These include Akın (2009, 2012), Sarafidis (2006) and Strotz (1956).

More precisely:

- At $t = 0$, a contract, which is a wage scheme, is offered to the agent by the principal. Then the agent accepts or rejects. If she rejects, the game ends and both the principal and the agent get zero utility.[7] If she accepts, they move on to the next period.

- At $t = 1$, the agent chooses an effort level, e_1, which is not observed by the principal. The output, q_1, is realized which is observable by both the agent and the principal. The wage payment, $w_1(q_1)$, is made to the agent.

- At $t = 2$, the agent chooses an effort level, e_2, which is not observed by the principal. The output, q_2, is realized which is observable by both the agent and the principal. The wage payment, $w_2(q_1, q_2)$, is made to the agent.

In the contracting stage, $t = 0$, there is no effort decision. The only decision made by the agent is to accept or reject the contract offered by the principal. We can think of this as getting a job offer in March but starting in September, just as most economics Ph.D. candidates experience. Thus we have three periods with two effort decisions. I also assume that once the contract is accepted at $t = 0$, both agent and principal are committed to the contract until the end of period 2. That is, I focus on long-term contracts and abstract from renegotiation issues.

2.2 Agent

There are two effort levels, 0 and 1, and two outcomes, q_h and q_l with $q_h > q_l$. The agent receives utility $u(w)$ from the wage w, and disutility $\psi(e)$ from exerting effort e. He is risk-averse, that is, $u' > 0$, $u'' < 0$. The disutility from exerting effort is given by $\psi(1) = \psi$ and $\psi(0) = 0$, where $\psi > 0$. The agent has additively separable preferences, so, his net utility in period $t \in \{1, 2\}$ is given by $v_t = u(w_t) - \psi(e_t)$.

The agent's present value of a flow of future utilities as of period t will be

$$v_t + \beta \sum_{s=t+1} \delta^{s-t} v_s$$

The discount factor between the current period and the next period is $\beta\delta$, but the discount factor between two adjacent periods in the future is δ. The agent is *time-consistent*

[7] To justify this zero-outside option assumption, assume that the outside option has also a lag in payment. Then it would be natural to normalize the outside option utility to zero.

(exponential discounter) when $\beta = 1$, and *time-inconsistent* (quasi-hyperbolic discounter) when $\beta < 1$. A time-inconsistent agent can be fully aware, partially aware or fully unaware of his time inconsistency. Denote the agent's belief about his true β by $\widehat{\beta}$. As in the literature, a time-inconsistent agent is *sophisticated* when he is fully aware of his inconsistency, that is, when $\widehat{\beta} = \beta < 1$. The agent is *partially naive* when $\beta < \widehat{\beta} < 1$ and *fully naive* when $\beta < \widehat{\beta} = 1$. The agent is *naive* when $\beta < \widehat{\beta} \leq 1$.

I focus on the case where the time-inconsistent agent is sophisticated. I consider the case where he is naive in Yılmaz (2013) and here I briefly discuss the results in Section 4.

Denote the probability of getting *high output* under *high effort,* $e = 1$ by $\Pr(q = q_h | e = 1) = p_1$ and the probability of getting *high output* under *low effort,* $e = 0$ by $\Pr(q = q_h | e = 0) = p_0$, where I assume $p_1 > p_0$.

There is no lending or borrowing. So the agent spends whatever wage he earns in a period within that period. I also assume away limited liability. However, I discuss the case with access to the credit market in Section 4 and provide an analysis in Appendix B.

2.3 Principal

The principal is risk-neutral, time-consistent and has a discount factor, δ_P. She cannot observe the effort levels exerted by the agent as in the standard moral hazard problem. However, she knows what type of agent she is facing. That is, she knows whether the agent is time-consistent or time-inconsistent.[8] This can be justified by considering a gym owner who faces customers who have present-biased preferences. It is possible that the gym owner has data on the accepted membership contracts and the actual usage of the gym by the members, and thus may have a better idea about the agent's inconsistency than the agent himself. Moreover, there is an asymmetric information regarding the effort level the agent picks, thus, before further complicating the model with an extra layer of information asymmetry, it is worth to look at the case of complete information regarding the inconsistency of the agent. Also, it is essential to study what the optimal contract would be when the agent is sophisticated and the principal knows it, before introducing the information asymmetry on this dimension.

I assume that the principal wants to implement high effort in each period. Specifically, I assume $q_h - q_l \geq \bar{t}$, where $\bar{t}$ is a threshold that is a function of model parameters.[9] This

[8] This assumption is also present in, for instance, DellaVigna and Malmendier (2004) and O'Donoghue and Rabin (1999b).

[9] Here, $\bar{t} = \frac{1+\delta_P}{(p_1-p_0)\delta_P}[p_1 u^{-1}(\psi + (1-p_1)\frac{\psi}{p_1-p_0}) + (1-p_1)u^{-1}(\psi - p_1\frac{\psi}{p_1-p_0})]$. See Lemma 2 in Appendix A for the details regarding this threshold.

ensures that it is optimal for the principal to implement high effort in each period.

The principal's problem is to find an individually rational and incentive compatible contract which specifies wages in both periods for all possible output realizations. That is, she maximizes her expected profit subject to individual rationality and incentive compatibility, over all possible contracts, $\{w_i, w_{ij}\}_{i,j\in\{h,l\}}$, where w_i is the wage paid in the first period if the output is q_i in the first period, and w_{ij} is the wage paid in the second period if the first period output is q_i and the second period output is q_j. Her payoff from a contract, $\{(w_h, w_l), (w_{hh}, w_{hl}, w_{lh}, w_{ll})\}$, given that it is accepted and the agent exerts high effort in both periods is given by

$$\begin{aligned}&p_1[q_h - w_h + \delta_P[p_1(q_h - w_{hh}) + (1-p_1)(q_l - w_{hl})]]\\&+(1-p_1)[q_l - w_l + \delta_P[p_1(q_h - w_{lh}) + (1-p_1)(q_l - w_{ll})]]\end{aligned}$$

3 Optimal Contract for a Sophisticated Agent

The principal wants to implement high effort, $e = 1$, in both periods. Denoting the utilities of the agent from wages with $u(w_{q_i}) = u_i$, and $u(w_{q_iq_j}) = u_{ij}$ where $i, j \in \{h, l\}$, and denoting the inverse of the utility function by $h(u)$, assuming it's continuously differentiable, the principal's problem when she is facing a sophisticated agent is

$$\begin{aligned}&\max_{\{u_i,u_{ij}\}_{i,j\in\{h,l\}}} p_1[q_h - h(u_h) + \delta_P[p_1(q_h - h(u_{hh})) + (1-p_1)(q_l - h(u_{hl}))]]\\&+(1-p_1)[q_l - h(u_l) + \delta_P[p_1(q_h - h(u_{lh})) + (1-p_1)(q_l - h(u_{ll}))]]\end{aligned}$$

subject to individual rationality, IR, and the incentive compatibility conditions, IC_1 and IC_2, for the two periods. This problem is equivalent to minimizing the cost of implementing high effort in both periods. That is,

$$\min_{\{u_i,u_{ij}\}_{i,j\in\{h,l\}}} p_1[h(u_h)+\delta_P[p_1h(u_{hh})+(1-p_1)h(u_{hl})]]+(1-p_1)[h(u_l)+\delta_P[p_1h(u_{lh})+(1-p_1)h(u_{ll})]]$$

subject to the individual rationality, IR, and the incentive compatibility conditions, IC_1 and IC_2, for the two periods.

Start with the second period incentive compatibility constraint, IC_2. Given the output realization in the first period, IC_2 ensures that the agent exerts high effort in the second period. IC_2 is given by

$$p_1 u_{ih} + (1-p_1)u_{il} - \psi \geq p_0 u_{ih} + (1-p_0)u_{il} \text{ for } i \in \{h,l\}$$

or

$$u_{ih} - u_{il} \geq \frac{\psi}{p_1 - p_0} \text{ for } i \in \{h,l\}$$

The first period incentive constraint, IC_1, that the agent will exert high effort in the second period, is given by

$$\begin{aligned} & p_1[u_h + \beta\delta(p_1 u_{hh} + (1-p_1)u_{hl} - \psi)] + (1-p_1)[u_l + \beta\delta(p_1 u_{lh} + (1-p_1)u_{ll} - \psi)] - \psi \\ \geq \; & p_0[u_h + \beta\delta(p_1 u_{hh} + (1-p_1)u_{hl} - \psi)] + (1-p_0)[u_l + \beta\delta(p_1 u_{lh} + (1-p_1)u_{ll} - \psi)] \end{aligned}$$

which can be written as

$$u_h + \beta\delta[p_1 u_{hh} + (1-p_1)u_{hl}] - u_l - \beta\delta[p_1 u_{lh} + (1-p_1)u_{ll}] \geq \frac{\psi}{p_1 - p_0}$$

Finally, the individual rationality constraint, IR, is given by

$$p_1[\beta\delta u_h + \beta\delta^2(p_1 u_{hh} + (1-p_1)u_{hl} - \psi)] + (1-p_1)[\beta\delta u_l + \beta\delta^2(p_1 u_{lh} + (1-p_1)u_{ll} - \psi)] - \psi \geq 0$$

or

$$p_1[u_h + \delta(p_1 u_{hh} + (1-p_1)u_{hl})] + (1-p_1)[u_l + \delta(p_1 u_{lh} + (1-p_1)u_{ll})] \geq (1+\delta)\psi$$

Observe that β enters IC_1 but it does not enter IR or IC_2. There is no payment made at $t = 0$. So from the point of view of $t = 0$, periods 1 and 2 are effectively discounted to the contracting stage at 1 and δ, respectively. However, agent, being sophisticated, knows that at $t = 1$ he will discount period 2 at $\beta\delta$. Hence in IC_1, periods 1 and 2 are discounted at 1 and $\beta\delta$, respectively.

The principal's problem becomes

$$\min_{\{u_i, u_{ij}\}_{i,j \in \{h,l\}}} p_1[h(u_h) + \delta_P\{p_1 h(u_{hh}) + (1-p_1)h(u_{hl})\}] + (1-p_1)[h(u_l) + \delta_P\{p_1 h(u_{lh}) + (1-p_1)h(u_{ll})\}]$$

subject to

$$
\begin{array}{ll}
(IR) & p_1[u_h + \delta(p_1 u_{hh} + (1-p_1)u_{hl})] + (1-p_1)[u_l + \delta(p_1 u_{lh} + (1-p_1)u_{ll})] \geq (1+\delta)\psi \\
(IC_1) & u_h + \beta\delta[p_1 u_{hh} + (1-p_1)u_{hl}] - u_l - \beta\delta[p_1 u_{lh} + (1-p_1)u_{ll}] \geq \frac{\psi}{p_1-p_0} \\
(IC_2) & u_{ih} - u_{il} \geq \frac{\psi}{p_1-p_0} \text{ for } i \in \{h,l\}
\end{array}
$$

For a given first period output level $q_i \in \{q_h, q_l\}$, the agent's continuation payoff will be $p_1 u_{ih} + (1-p_1)u_{il} - \psi$. If the agent has been promised Eu_i for the second period when the first period output realization is q_i, then u_{ih} and u_{il} are defined to be

$$p_1 u_{ih} + (1-p_1)u_{il} - \psi = Eu_i \text{ for } i \in \{h,l\}$$

Once the principal promises the agent a utility of Eu_i, the continuation of the optimal contract for the second period will be given by the solution to the following problem

$$\min_{u_{ih}, u_{il}} p_1 h(u_{ih}) + (1-p_1)h(u_{il})$$

subject to

$$
\begin{aligned}
u_{ih} - u_{il} &\geq \frac{\psi}{p_1 - p_0} \\
p_1 u_{ih} + (1-p_1)u_{il} - \psi &\geq Eu_i
\end{aligned}
$$

This is a static problem and it is straightforward to show that both constraints bind. Hence, for a given first period output, q_i, the second period payoffs to the agent are

$$
\begin{aligned}
u_{ih} &= \psi + Eu_i + (1-p_1)\frac{\psi}{p_1 - p_0} \\
u_{il} &= \psi + Eu_i - p_1\frac{\psi}{p_1 - p_0}
\end{aligned}
$$

Denote the cost of implementing the high effort level in the second period, given that the promised second period utility is Eu_i, by $C_2(Eu_i)$. Then,

$$C_2(Eu_i) = p_1 h(u_{ih}) + (1-p_1)h(u_{il})$$

Now the principal's problem can be reduced to

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(Eu_h) + (1-p_1)C_2(Eu_l)]$$

subject to

$$\begin{array}{ll} (IR) & p_1[u_h + \delta Eu_h] + (1-p_1)[u_l + \delta Eu_l] \geq \psi \\ (IC) & u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0} \end{array}$$

Note that incentives in the first period depend on the second period only through Eu_i, not through u_{ih} or u_{il}. Also, note that $h(\cdot)$ is a strictly increasing and strictly convex function because it is the inverse of a strictly increasing and strictly concave function $u(\cdot)$. $C_2(\cdot)$ is also a strictly increasing and strictly convex function.[10] Thus, the first order conditions will be both necessary and sufficient. $C_2(\cdot)$ is also continuously differentiable.[11]

Now, attaching λ to IR and μ to IC_1 we have the following first order conditions with respect to u_h, u_l, Eu_h and Eu_l, respectively.

$$h'(u_h) = \frac{\mu}{p_1} + \lambda \tag{1}$$

$$h'(u_l) = -\frac{\mu}{1-p_1} + \lambda \tag{2}$$

$$\frac{\delta_P}{\delta} C_2'(Eu_h) = \frac{\beta\mu}{p_1} + \lambda \tag{3}$$

$$\frac{\delta_P}{\delta} C_2'(Eu_l) = -\frac{\beta\mu}{1-p_1} + \lambda \tag{4}$$

[10] $C_2(Eu_i) = p_1 h(u_{ih}) + (1-p_1)h(u_{il}) = p_1 h(\psi + Eu_i + \frac{(1-p_1)\psi}{p_1 - p_0}) + (1-p_1)h(\psi + Eu_i - \frac{p_1\psi}{p_1-p_0})$. Since $h(\cdot)$ is strictly increasing, C_2 is clearly strictly increasing. Also, since $h(\cdot)$ is strictly convex, C_2 is also strictly convex: $C_2'(.) = p_1 h'(.) + (1-p_1)h'(.)$, and since $h'' > 0$, we also have $C_2'' > 0$.

[11] For C_2, the sufficient conditions for continuous differentiability in Benveniste and Scheinkman (1979) are satisfied: The constraint set is convex since both constraints are linear, and it has a nonempty interior. The function $h(\cdot)$ is strictly convex and differentiable. Given Eu_i^0, an optimal solution exists (the one defined above which is in the interior of the constraint set) and $C_2(Eu_i)$ is well defined for Eu_i in some neighborhood of Eu_i^0, since $h(\cdot)$ is well defined. Also, given Eu_i, let $u_{ih}^*(Eu_i)$ and $u_{ih}^*(Eu_i)$ solve the second period cost minimization problem where the principal implements high effort. Then, the minimized cost is $C_2(Eu_i) = p_1 h(u_{ih}^*(Eu_i)) + (1-p_1)h(u_{il}^*(Eu_i))$. By Envelope Theorem, we have

$$\frac{dC_2}{dEu_i} = \frac{\partial[p_1 h(u_{ih}^*(Eu_i)) + (1-p_1)h(u_{il}^*(Eu_i))]}{\partial Eu_i} + \phi^*$$

where ϕ^* is the Lagrange multiplier attached to the incentive compatibility condition in the second period minimization problem, given Eu_i (note that Eu_i does not enter the individual rationality constraint).

(1) and (2) imply that

$$\lambda = p_1 h'(u_h) + (1-p_1)h'(u_l) \tag{5}$$

$$\mu = p_1(1-p_1)(h'(u_h) - h'(u_l)) \tag{6}$$

(1) and (3) imply that

$$h'(u_h) = \frac{\delta_P}{\delta} C_2'(Eu_h) + \frac{\mu}{p_1}(1-\beta) \tag{7}$$

(2) and (4) imply that

$$h'(u_l) = \frac{\delta_P}{\delta} C_2'(Eu_l) - \frac{\mu}{1-p_1}(1-\beta) \tag{8}$$

Lemma 1 $\lambda > 0$ *and* $\mu > 0$.

Proof. $\lambda > 0$ follows immediately from (5). By construction $\mu \geq 0$. Note that $\mu = 0$ implies $u_h = u_l$ and $Eu_h = Eu_l$. But then IC of the reduced problem is violated. ■

Henceforth, I will denote a time-consistent agent who has discount factor δ with TC_δ and a sophisticated time-inconsistent agent who has discounting function represented by $(1, \beta\delta, \beta\delta^2)$ with SO. I will also use TC_δ and SO as superscripts whenever appropriate.

If the agent is time-consistent, then the conditions (7) and (8) correspond to the relationship between contingent wages across periods presented in Rogerson (1985). That is, if $\beta = 1$, then conditions (7) and (8) imply $h'(u_i^{TC_\delta}) = \frac{\delta_P}{\delta} C_2'(Eu_i^{TC_\delta})$ for $i \in \{h, l\}$. The definition of C_2 implies that

$$C_2'(Eu_i^{TC_\delta}) = p_1 h'(u_{ih}^{TC_\delta}) + (1-p_1)h'(u_{il}^{TC_\delta}) = E_{q_j}(h'(u_{ij}^{TC_\delta}))$$

Therefore, for a time-consistent agent with discount factor δ, we have

$$h'(u_i^{TC_\delta}) = \frac{\delta_P}{\delta} E_j(h'(u_{ij}^{TC_\delta})) \text{ for } i \in \{h, l\} \tag{9}$$

This relationship is often referred to as the *martingale property.* This property says that the principal intertemporally smooths the agent's rewards over time in a way that the cost of promising one more unit of utility today is exactly equal to the gain, discounted appropriately, from having one less unit of utility to promise tomorrow, following any realization of the output. That is, the marginal cost of rewarding the agent in the first period for an output realization of q_i must be equal to marginal benefit of delaying these awards, discounted appropriately, in the continuation of the contract given that the first period output is q_i. The principal intertemporally spreads the rewards to the agent to minimize the cost of implementing high effort in the first period.

Now, consider a time-consistent agent with a discount factor $\beta\delta$, with $\beta < 1$. Then (9) implies

$$\begin{aligned} h'(u_h^{TC_{\beta\delta}}) &= \frac{\delta_P}{\beta\delta} E_{q_j}(h'(u_{hq_j}^{TC_{\beta\delta}})) \\ h'(u_l^{TC_{\beta\delta}}) &= \frac{\delta_P}{\beta\delta} E_{q_j}(h'(u_{lq_j}^{TC_{\beta\delta}})) \end{aligned}$$

Put differently,

$$\begin{aligned} h'(u_h^{TC_{\beta\delta}}) &> \frac{\delta_P}{\delta} E_{q_j}(h'(u_{hq_j}^{TC_{\beta\delta}})) \\ h'(u_l^{TC_{\beta\delta}}) &> \frac{\delta_P}{\delta} E_{q_j}(h'(u_{lq_j}^{TC_{\beta\delta}})) \end{aligned}$$

So for a time-consistent agent, the principal needs to shift the payments to the first period for both output realizations, as the agent gets more impatient.[12] However shifting payments to the first period for SO is not as easy as it is for $TC_{\beta\delta}$. Both $TC_{\beta\delta}$ and SO have the same IC_1. But for $TC_{\beta\delta}$, IR uses $(1, \beta\delta)$ discounting, whereas IR for SO uses $(1, \delta)$ discounting. That is, for SO the second period is more important than it is for $TC_{\beta\delta}$ which makes it harder for the principal to shift payments to the first period for *both* realizations. This intuition suggests that if the principal shifts the payments to the first period for one output realization, then for the other output realization, she must shift the payments to the second period. Now, we can show that the martingale property does not hold for a sophisticated agent.

Proposition 1 *The martingale property fails when the agent is sophisticated. More precisely,*

$$\begin{aligned} h'(u_h^{SO}) &> \frac{\delta_P}{\delta} E_j(h'(u_{hj}^{SO})) \\ h'(u_l^{SO}) &< \frac{\delta_P}{\delta} E_j(h'(u_{lj}^{SO})) \end{aligned}$$

Proof. Since $\mu > 0$, (7) and (8) imply $h'(u_h) > \frac{\delta_P}{\delta} C_2'(Eu_h)$ and $h'(u_l) < \frac{\delta_P}{\delta} C_2'(Eu_l)$.

[12] This is because $h'' > 0$.

By definition $C_2'(Eu_i) = E_j(h'(u_{ij}))$. Hence $\delta h'(u_h^{SO}) > \delta_P E_j(h'(u_{hj}^{SO}))$ and $C_2'(Eu_l) = E_{q_j}(h'(u_{lq_j}))$. ■

Suppose $\delta_P = \delta$. Then this proposition says that the marginal cost of rewarding the sophisticated agent for a high level of output in the first period is higher than the expected marginal cost of promising these awards in the second period given that the first period output level is high and symmetrically when the first period output level is low.

Using the above result, I compare the first period utility for a time-consistent agent with discount factor δ to that for a sophisticated time-inconsistent agent with $(1, \beta\delta, \beta\delta^2)$ in the optimal contract that implements high effort in both periods.

Proposition 2 *Assume $\delta_P = \delta$. Then $u_h^{SO} > u_h^{TC_\delta}$ and $u_l^{SO} < u_l^{TC_\delta}$.*

Proof. See Appendix A. ■

For a time-inconsistent sophisticated agent with discounting $(1, \beta\delta, \beta\delta^2)$, the principal should increase the wage for good performance and decrease it for bad performance, relative to the wage scheme for a time-consistent agent with a discount factor δ. That is, it's optimal for the principal to increase the risk the sophisticated time-inconsistent agent faces in the first period relative to the time-consistent agent. The intuition for this result is the following. First set $\delta = 1$ for simplicity. Denote the wage differential and expected wage differential with $\triangle u = u_h - u_l$ and $\triangle EU = EU_h - EU_l$, respectively. Now suppose that $\{u_i, EU_j\}_{i,j \in \{h,l\}}$ is the optimal contract for a time-consistent agent with discount factor $\delta = 1$. Then consider the following two options for the sophisticated agent. In the first option increase $\triangle u$ by $\triangle_1$ and keep $\triangle EU$ the same. In the second option, increase $\triangle EU$ by $\triangle_2$ and keep $\triangle u$ the same. If both options provide the agent with the same incentives, it should be the case that $\triangle_1 = \beta \triangle_2$. This is because the sophisticated agent discounts the second period to the first period by a factor $\beta\delta$ which is β, since $\delta = 1$. Hence, $\triangle_1 < \triangle_2$. Sophisticated agent's individual rationality constraint will hold with slack under the first option if it binds under the second option. This is because $\beta\triangle_1 < \beta \triangle_2$. Then, the principal can increase $\triangle u$ by $\triangle_2$, and hence provide more incentives and still not violate the individual rationality.

Now I compare the expected profits that the principal achieves in the optimal contracts with a time-consistent agent and with a sophisticated time-inconsistent agent. However, there are two sensible comparisons. I will compare the expected profit from SO to the expected profit from TC_δ, and to that from $TC_{\beta\delta}$. The proposition below compares the expected profit from SO to the expected profit from TC_δ.

Proposition 3 *The expected profit from TC_δ is higher than that from SO.*

Proof. Recall the optimization problem when the agent is time-inconsistent and sophisticated.

$$\min_{u_i, Eu_i} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(Eu_h) + (1-p_1)C_2(Eu_l)]$$

subject to

$$\begin{array}{ll} (IR) & p_1[u_h + \delta Eu_h] + (1-p_1)[u_l + \delta Eu_l] \geq \psi \\ (IC) & \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0} \end{array}$$

The solution $\{u_h^*, u_l^*, EU_h^*, EU_l^*\}$ and the Lagrange multipliers are continuously differentiable functions of β. Also the non-degenerate constraint qualification holds.[13] Hence by the envelope theorem, we get $\frac{dE\pi(\beta)}{d\beta} = \mu\delta(Eu_h - Eu_l)$ where μ is the Lagrange multiplier attached to the IC. Both μ and δ are positive. Note that, $Eu_h > Eu_l$, which follows directly from equations (3) and (4) and the fact that $\mu > 0$ and C_2' is a strictly increasing function. Thus, $\frac{dE\pi(\beta)}{d\beta} > 0$. Hence, $E\pi(\beta = 1) > E\pi(\beta < 1)$. ■

This result is driven by the fact that the sophisticated time-inconsistent agent discounts the second period more than the consistent agent does when incentives are considered. Hence the effect of the second period on the first period incentives is lower for SO compared to TC_δ. But both agents discount the second period the same when the contract is evaluated in the contracting stage. Therefore it's harder to implement high effort with SO, which gives rise to lower profits.

A comparison between the expected profit from SO to that from $TC_{\beta\delta}$ is given in the following proposition.

Proposition 4 *The expected profit from $TC_{\beta\delta}$ is lower(higher) than that from SO, whenever the expected second period promised utility, $p_1 Eu_h + (1-p_1)Eu_l$, is positive for $TC_{\beta\delta}$(negative for SO).*

Proof. The individual rationality and incentive constraints are as follows

$$\begin{array}{ll} (IR) & p_1 u_h + (1-p_1)u_l + \alpha\delta[p_1 Eu_h + (1-p_1)Eu_l] \geq \psi \\ (IC) & \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0} \end{array}$$

where $\alpha = 1$ represents SO and $\alpha = \beta$ represents $TC_{\beta\delta}$. We get $\frac{dE\pi(\alpha)}{d\alpha} = \lambda(p_1 Eu_h + (1-p_1)Eu_l)$ where λ is the Lagrange multiplier attached to the individual rationality

[13]The rank of the augmented matrix of the constraint system is 2.

constraint. Note that α does not enter the objective function. If $p_1 Eu_h + (1-p_1)Eu_l > 0$ at $\alpha = \beta$, then $E\pi(\alpha = \beta) < E\pi(\alpha = 1)$. That is, the principal prefers SO to $TC_{\beta\delta}$. Likewise, if $p_1 Eu_h + (1-p_1)Eu_l < 0$ at $\alpha = 1$, then $E\pi(\alpha = \beta) > E\pi(\alpha = 1)$, so the principal prefers $TC_{\beta\delta}$ to SO. ■

Whenever the optimal contract promises the time-consistent agent with a discount factor $\beta\delta$ a positive second period expected utility, then it is less costly to implement high effort with a sophisticated agent who has a discounting function given by $(1, \beta\delta, \beta\delta^2)$. This is because the contract for the consistent agent will be accepted by the inconsistent agent too. That is, the individual rationality constraint will hold for an inconsistent agent with slack. Then the principal can alter the wage scheme by appropriately reducing payments for high and low outputs without violating the incentive constraint. Note that both agents have the same incentive constraint. Hence, the principal can do better with SO than with $TC_{\beta\delta}$, when optimal contract for $TC_{\beta\delta}$ promises him a positive second period expected utility. A positive second period expected utility is an indication of the fact that it is relatively harder to implement high effort in both periods. When it is negative for the sophisticated agent, then the principal can do better with $TC_{\beta\delta}$, because $TC_{\beta\delta}$ will value the second period less than SO from the contracting stage point of view. Hence the contract for SO will make the $TC_{\beta\delta}$ to exert high effort, but with slack in the individually rationality constraint. Therefore, the principal facing $TC_{\beta\delta}$ can implement high effort with lower wages when the second period expected utility is negative for SO.

4 Discussion and Extensions

In this paper, I analyzed a repeated moral hazard problem with a time-inconsistent agent. To capture time-inconsistency, I assumed *$\beta\delta$-preferences* which is simple enough and widely used in the literature. I posed the following question: Would a risk neutral principal prefer to face a time-consistent agent or a time-inconsistent agent? Assuming that the time-inconsistent agent is fully aware of his inconsistency, I showed that the optimal contract for the time-inconsistent agent, relative to the optimal contract for the time-consistent agent, has a higher reward for high performance and a lower reward when the output is low, in the first period. The principal is better off facing a time-consistent agent with a discount factor δ, than facing a sophisticated time-inconsistent agent with discounting $(1, \beta\delta, \beta\delta^2)$. She is worse off facing a time-consistent agent with a discount factor $\beta\delta$, than facing a sophisticated time-inconsistent agent with discounting $(1, \beta\delta, \beta\delta^2)$

if the promised second period expected utility is positive for the consistent agent.

There is a number of extensions one might consider. Here, I will first analyse 6 possible extensions and provide the details in Appendix B. Then, I will also discuss some open problems.

1. Continuous Effort and $N > 2$ Outcome Levels:

One possible extension is the one with the continuous effort levels as in Rogerson (1985). Suppose, the effort choice is $e \in [0, 1]$ and there are $N > 2$ many possible outcome levels. It turns out that under a nontriviality condition, the Proposition 1 still holds. The condition requires that $p_k(e)$ is not constant at $e = e^*$ for some $k \in N$, where the principal is implementing $e^* \in (0, 1)$. In terms of evolution of wages, the result in Rogerson (1985) is partly still valid: whenever $\frac{1}{u'}$ is convex, it is still true that the period 1 wage is greater than the expected period 2 wage for those outcomes with $p_i'(e = e^*) > 0$. It is not necessarily true for those outcomes with $p_i'(e = e^*) < 0$, though. So for those period 1 outcomes that are relatively better, it is still optimal to reward the agent more today. But for the other outcomes it is optimal to punish the agent more today. This is very much parallel to the Proposition 2 in the text, where the sophisticated agent is awarded more if the outcome is good and punished more if the outcome is bad, all relative to a time-consistent agent. The analysis with all the details is given in Appendix B.

2. Effort Choice with $T \geq 3$ periods:

In the model, I assumed that the effort choice was made in only two periods, although there were three periods in total. This way, the individual rationality constraint became independent of β. One extension is to allow for efforts choice to be made in all periods with $T \geq 3$. In Appendix B, I show that the critical equations (7) and (8) which lead to Proposition 1 are still satisfied, when the second and the third periods are considered. Thus the martingale property will still fail under this alternative model.

3. Access to the Credit Market:

I also consider the possibility of saving and borrowing. Suppose the agent is allowed to have access to the credit market right after the outcome is realized at the end of period 1 ($t = 1$) and can save or borrow at the rate r. It turns out that a sophisticated time-insonsistent agent *may* want to borrow, unlike the consistent agent in Rogerson (1985). If $\beta = 1$, then the agent always saves, thus Rogerson (1985) is captured as a special case. For low values of β however the agent has stronger present biased preferences hence is willing to borrow more today, rather than saving. The details are in Appendix B.

4. Comparison with an Alternative Time-consistent Agent:

One other sensible time-consistent benchmark is the one with a discount factor Δ such that $\delta\beta + \delta^2\beta = \Delta + \Delta^2$. That is, the time-inconsistent agent and the time-consistent agent have the same average discount factor. I show that for the principal it is more costly to implement high effort (in each period) with a sophisticated agent than it is with a time-consistent agent who has a discount factor Δ, whenever the expected second period promised utility for the time-consistent agent is positive and a condition on δ and β parameters is satisfied. The result is also true whenever the expected second period promised utility for the sophisticated agent is negative. The proof is in Appendix B.

The intuition for this result is as follows. The individual rationality of the sophisticated agent will hold with a slack when using the contract for the time-consistent agent with Δ, since $\delta > \Delta$. But the incentive compatibility constraint of sophisticated agent will not hold anymore since $\Delta > \delta\beta$. Whenever, the sophisticated agent's inconsistency is strong (a lower β), the principal will need to provide more incentives for the sophisticated agent (relative to the time-consistent agent). Thus, for the time-consistent agent, Eu_h and Eu_l will be higher and $Eu_h - Eu_l$ will be lower than those for the sophisticated agent. Since it's better for the principal to postpone the payments while providing the necessary incentives, it's less costly to implement the high effort (in each period) with a time-consistent agent with Δ than with a sophisticated agent. Whenever, the second period promised expected utility for the sophisticated agent is negative, the optimal contract for the sophisticated agent will satisfy the IR and IC constraints for the time-consistent agent, both with a slack. Thus, it will again be less costly to implement the high effort (in each period) with a time-consistent agent with Δ than a sophisticated agent.

5. Relaxing the Full Commitment Assumption:

In the model, I assumed that the agent fully commits to the contract when he accepts it in the contracting stage. Here I discuss what could happen if we assume away full commitment by the agent. It turns out that we have the following property at the optimal contract:

$$p_1 h'(u_h) + (1-p_1)h'(u_l) < \frac{\delta_P}{\delta}(p_1 C_2'(Eu_h) + (1-p_1)C_2'(Eu_l))$$

and

$$h'(u_l) < \frac{\delta_P}{\delta} C_2'(Eu_l)$$

But we cannot rule out $h'(u_h) \geq \frac{\delta_P}{\delta} C_2'(Eu_h)$ or $h'(u_h) < \frac{\delta_P}{\delta} C_2'(Eu_h)$. The first case is not interesting because in that case the marginal cost of rewarding the agent in the second period is relatively lower hence it is easier to keep the agent in the contract. Thus, the results would not change in that case. However, if it is the other case, then it means for both output realizations it is more costly to promise the agent rewards in the second period. But to keep the agent in the contract the second period payments should be higher, thus it is more costly to both keep the agent in the contract and also give the right incentives. For the analysis please see Appendix B.

6. Naive Agent:

With a naive agent the principal's problem is more involved because she can deceive the agent and potentially get information rents. In Yılmaz (2013), I show that the principal, implementing high effort in both periods, is indifferent between facing a naive agent and facing a sophisticated agent. This is particularly striking because with a naive time-inconsistent agent the principal can choose to deceive the agent. However, such an opportunity to manipulate does not provide the principal with higher profits and in the optimal contract the principal chooses not to deceive the agent at all. For the detailed analysis, please see Yılmaz (2013).

Now, I discuss some open questions.

Long-Term Contracts vs Short-Term Contracts: In the model, I focused on long-term contracts only. However, it's important to discuss the model and it's implications by reconsidering the relationship between long-term and short-term contracts. We know that, when agents are time-consistent, long-term contracts weakly Pareto dominate short-term contracts: principal can always offer the payment scheme of these short-term contracts in a long-term contract. For a sophisticated time-inconsistent agent whose preferences are captured through $\beta - \delta$ discounting scheme, the agent is fully aware of his inconsistency. Thus, from today's perspective, when picking an effort level tomorrow, he knows his self tomorrow may not agree with him. To force his self tomorrow to do the right thing, a commitment device will potentially be valuable for the agent. Thus, intuitively, a long-term contract will be preferred by a time-inconsistent agent if it is preferred by a time-consistent agent. So, when facing a time-consistent agent, if it is optimal to offer a long-term contract rather than a sequence of short term contracts, then it will still be optimal to do so when facing a sophisticated time-inconsistent agent.

Fudenberg, Holmstrom, Milgrom (1990) show, under certain assumptions,[14] the efficient long-term contract can be implemented by a sequence of short-term contracts, hence commitments are not necessary. I believe by considering time-inconsistent agents in their setup and checking whether the result is still valid is important and can be another paper

[14]These conditions include no adverse selection at the time of renegotiation (the preferences of the principal and the agent over future contingent outcomes and future technological opportunities must be common knowledge), payments are based on the joint information with no delay, and the agent's preferences are additively separable

idea on its own. I believe there is a chance that for a sophisticated time-inconsistent agent, the result may not be valid, relying on the intuition I provided above, namely, commitment may be valuable for a sophisticated time-inconsistent agent.

Renegotiation: In the model, I assumed away any renegotiation issues. However, it could be the case that the two parties renegotiate the terms of the contract and induce a Pareto-improvement through the course of interaction. One question is, when the agent is a sophisticated time-inconsistent agent, is it harder or easier to get renegotiation-proof contracts, compared with the case of a time-consistent agent? Intuitively, commitment is valuable for the sophisticated agent at the contracting stage because it prevents the future selves from renegotiating with the principal and creating a new contract that's worse for the self today. So, this potentially makes it easier to get renegotiation-proof contracts. However, from the principal's perspective, it might be better to renegotiate in the future with the future self, since when contracting with the self today under full commitment, the principal has to provide incentives for the future self as well. But, renegotiating with each self at the relevant period might be easier to generate ex post efficiency. Thus, I believe, the answer depends on which one of these two forces dominates the other.

5 Appendix A: Proofs

Lemma 2 *If* $q_h - q_l \geq \frac{1+\delta_P}{(p_1-p_0)\delta_P}[p_1 u^{-1}(\psi + (1-p_1)\frac{\psi}{p_1-p_0}) + (1-p_1)u^{-1}(\psi - p_1\frac{\psi}{p_1-p_0})]$*, then it is optimal for the principal to implement high effort in both periods.*

Proof. Let $V(e_1, e_2)$ be the maximum profit level principal gets when she implements efforts e_1 and e_2 in period 1 and period 2, respectively. For implementing high effort in both periods to be optimal we need, $V(1,1) \geq V(1,0)$, $V(1,1) \geq V(0,1)$ and $V(1,1) \geq V(0,0)$. Let $\pi_1 = p_1 q_h + (1-p_1)q_l$ and $\pi_0 = p_0 q_h + (1-p_0)q_l$. Thus, $\pi_1 - \pi_0 = (p_1 - p_0)(q_h - q_l)$. Also, let $h = u^{-1}$, as in the text.

Principal implementing effort schedule (0,0): The optimal wage scheme that implements $e = 0$ in both periods is clearly $w_i = w_{ij} = 0$ for each $i, j \in \{h, l\}$. Thus, $V(0,0) = (1+\delta_P)\pi_0$.

Principal implementing effort schedule (0,1): If the agent has been promised Eu_i for the second period when the first period output realization is q_i, then the principal's problem can be reduced to

$$\min_{\{u_i, Eu_i\}_{i \in \{h,l\}}} p_0 h(u_h) + (1-p_0)h(u_l) + \delta_P[p_0 C_2(Eu_h) + (1-p_0)C_2(Eu_l)]$$

subject to

$$p_0[u_h + \delta Eu_h] + (1-p_0)[u_l + \delta Eu_l] \geq 0 \qquad \text{and} \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq 0$$

where $C_2(Eu_i)$ denotes the cost of implementing the high effort level in the second period, given that the promised second period utility is Eu_i, that is, $C_2(Eu_i) = p_1 h(u_{ih}(Eu_i)) + (1-p_1)h(u_{il}(Eu_i))$, which is increasing in Eu_i. The solution for this problem is $u_i = Eu_i = 0$ for each $i \in \{h, l\}$. Thus, $V(0,1) = \pi_0 + \delta_P \pi_1 - \delta_P[p_0[p_1 h(u_{hh}) + (1-p_1)h(u_{hl})] + (1-p_0)[p_1 h(u_{lh}) + (1-p_1)h(u_{ll})]]$ where $\{u_{ij}\}_{i,j\in\{h,l\}}$ solve the following problem.

$$\min_{u_{ih},u_{il}} p_1 h(u_{ih}) + (1-p_1)h(u_{il})$$

subject to

$$u_{ih} - u_{il} \geq \frac{\psi}{p_1 - p_0} \qquad \text{and} \qquad p_1 u_{ih} + (1-p_1)u_{il} \geq \psi$$

The solution to this problem is given in the text. We simply plug $Eu_i = 0$ and get

$$\begin{aligned} u_{hh} = u_{lh} &= \psi + (1-p_1)\frac{\psi}{p_1 - p_0} \\ u_{hl} = u_{ll} &= \psi - p_1 \frac{\psi}{p_1 - p_0} \end{aligned}$$

Thus, $p_1 h(u_{hh}) + (1-p_1)h(u_{hl}) = p_1 h(u_{lh}) + (1-p_1)h(u_{ll})$. Let $p_1 h(u_{hh}) + (1-p_1)h(u_{hl}) = C(\psi, p_1, p_0)$. Thus, we get $V(0,1) = \pi_0 + \delta_P \pi_1 - \delta_P C(\psi, p_1, p_0)$, where $C(\psi, p_1, p_0) = p_1 u^{-1}(\psi + (1-p_1)\frac{\psi}{p_1-p_0}) + (1-p_1)u^{-1}(\psi - p_1\frac{\psi}{p_1-p_0})$.

Principal implementing effort schedule (1,0): Clearly, $\pi_1 + \delta_P \pi_0 > V(1,0)$.

Principal implementing effort schedule (1,1): As shown in the text, the maximum profit is $V(1,1) = \pi_1(1+\delta_P) - [p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(Eu_h) + (1-p_1)C_2(Eu_l)]]$ where $\{u_i\}_{i\in\{h,l\}}$ and $\{Eu_i\}_{i\in\{h,l\}}$ solve the following reduced problem

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(Eu_h) + (1-p_1)C_2(Eu_l)]$$

subject to

$$p_1[u_h + \delta Eu_h] + (1-p_1)[u_l + \delta Eu_l] \geq \psi \qquad \text{and} \qquad u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0}$$

If we restrict attention to $Eu_i = 0$ for both $i \in \{h, l\}$, then the solution to this restricted problem will give an overall profit that is less than or equal to $V(1,1)$. Thus, this restricted problem is

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(0) + (1-p_1)C_2(0)]$$

subject to

$$p_1 u_h + (1-p_1)u_l \geq \psi \qquad \text{and} \qquad u_h - u_l \geq \frac{\psi}{p_1 - p_0}$$

For a promised second period expected payoff, $Eu_i = 0$, to the agent, when the first period output realization is q_i, the problem in the second period is the same problem with the second period problem described and solved above under the case principal implementing (0,1). Thus, $C_2(0) = C(\psi, p_1, p_0)$ given above. But then the restricted problem is reduced to

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P C(\psi, p_1, p_0)$$

subject to

$$p_1 u_h + (1-p_1)u_l \geq \psi \qquad \text{and} \qquad u_h - u_l \geq \frac{\psi}{p_1 - p_0}$$

which is again equivalent to the one described above. Thus, $p_1 h(u_h) + (1-p_1)h(u_l) = C(\psi, p_1, p_0)$. Thus, we get $V(1,1) \geq \pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P)$.

Now, we put all these together:

(1) If $\pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P) \geq V(0,0)$, then $V(1,1) \geq V(0,0)$. Thus, we need $\pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P) \geq (1+\delta_P)\pi_0$, that is, $\pi_1 - \pi_0 \geq C(\psi, p_1, p_0)$.

(2) If $\pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P) \geq V(0,1)$, then $V(1,1) \geq V(0,1)$. Thus, we need $\pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P) \geq \pi_0 + \delta_P\pi_1 - \delta_P C(\psi, p_1, p_0)$, that is, $\pi_1 - \pi_0 \geq C(\psi, p_1, p_0)$.

(3) If $\pi_1(1+\delta_P) - C(\psi, p_1, p_0)(1+\delta_P) \geq \pi_1 + \delta_P\pi_0$, then $V(1,1) \geq V(1,0)$ since $\pi_1 + \delta_P\pi_0 > V(1,0)$. Thus, we need $\pi_1 - \pi_0 \geq \frac{1+\delta_P}{\delta_P} C(\psi, p_1, p_0)$.

Since $\frac{1+\delta_P}{\delta_P} > 1$, the three comparisons above imply that if $\pi_1 - \pi_0 \geq \frac{1+\delta_P}{\delta_P} C(\psi, p_1, p_0)$, that is, if $q_h - q_l \geq \frac{1+\delta_P}{\delta_P(p_1-p_0)}[p_1 u^{-1}(\psi + (1-p_1)\frac{\psi}{p_1-p_0}) + (1-p_1)u^{-1}(\psi - p_1\frac{\psi}{p_1-p_0})]$, then it will be optimal for the principal to implement the effort schedule $(1,1)$. ■

Proof of Proposition 2. Recall the IR and IC of the reduced problem for the sophisticated agent

$$(IR) \quad p_1[u_h^{SO} + \delta(Eu_h^{SO})] + (1-p_1)[u_l^{SO} + \delta(Eu_l^{SO})] \geq \psi$$

$$(IC) \qquad u_h^{SO} + \delta\beta[Eu_h^{SO}] - u_l - \delta\beta[Eu_l^{SO}] \geq \frac{\psi}{p_1 - p_0}$$

Multiplying IC with $1 - p_1$, and adding it up with IR we get

$$u_h^{SO} = \psi(1 + \frac{1-p_1}{p_1 - p_0}) - \delta[(\beta + (1-\beta)p_1)Eu_h^{SO} + (1 - (\beta + (1-\beta)p_1))Eu_l^{SO}]$$

Multiplying IC with p_1, and subtracting it from IR we get

$$u_l^{SO} = \psi(1 - \frac{p_1}{p_1 - p_0}) - \delta[(1-\beta)p_1 Eu_h^{SO} + (1 - (1-\beta)p_1)Eu_l^{SO}]$$

And corresponding utilities for the time-consistent agent are

$$\begin{aligned} u_h^{TC_\delta} &= \psi(1 + \frac{1-p_1}{p_1 - p_0}) - \delta Eu_h^{TC_\delta} \\ u_l^{TC_\delta} &= \psi(1 - \frac{p_1}{p_1 - p_0}) - \delta Eu_l^{TC_\delta} \end{aligned}$$

Hence

$$\begin{aligned} u_h^{SO} + \delta[(\beta + (1-\beta)p_1)Eu_h^{SO} + (1 - (\beta + (1-\beta)p_1))Eu_l^{SO}] &= u_h^{TC_\delta} + \delta Eu_h^{TC_\delta} \\ u_l^{SO} + \delta[(1-\beta)p_1 Eu_h^{SO} + (1 - (1-\beta)p_1)Eu_l^{SO}] &= u_l^{TC_\delta} + \delta Eu_l^{TC_\delta} \end{aligned}$$

Note that $1 > \beta + (1-\beta)p_1 > 0$ and $1 > (1-\beta)p_1 > 0$. Since $Eu_h^{SO} > Eu_l^{SO}$, we get

$$u_h^{SO} + \delta Eu_h^{SO} > u_h^{TC_\delta} + \delta Eu_h^{TC_\delta} \tag{10}$$

$$u_l^{SO} + \delta Eu_l^{SO} < u_l^{TC_\delta} + \delta Eu_l^{TC_\delta} \tag{11}$$

Now using Proposition 1, we have

$$\begin{aligned} h'(u_h^{SO}) &> C_2'(Eu_h^{SO}) \qquad & h'(u_h^{TC_\delta}) &= C_2'(Eu_h^{TC_\delta}) \\ h'(u_l^{SO}) &< C_2'(Eu_l^{SO}) \qquad & h'(u_l^{TC_\delta}) &= C_2'(Eu_l^{TC_\delta}) \end{aligned}$$

Define $g = (h')^{-1}$. Note that g and $C_2'(.)$ are strictly increasing functions.[15] Therefore

$$u_h^{SO} > (g \circ C_2')(Eu_h^{SO}) \qquad u_h^{TC_\delta} = (g \circ C_2')(Eu_h^{TC_\delta}) \tag{12}$$

$$u_l^{SO} < (g \circ C_2')(Eu_l^{SO}) \qquad u_l^{TC_\delta} = (g \circ C_2')(Eu_l^{TC_\delta}) \tag{13}$$

[15] $g'(.) = \frac{1}{h''(g(.))} > 0$ since $h'' > 0$. To see that $C_2'(.)$ is strictly increasing, see footnote 10.

where $g \circ C_2'$ is an increasing function. Now suppose $u_h^{SO} \leq u_h^{TC_\delta}$. Then (10) implies $Eu_h^{SO} > Eu_h^{TC_\delta}$, whereas (12) implies $(g \circ C_2')(Eu_h^{SO}) < (g \circ C_2')(Eu_h^{TC_\delta})$ which in turn implies $Eu_h^{SO} < Eu_h^{TC_\delta}$. Hence $u_h^{SO} > u_h^{TC_\delta}$. And $u_l^{SO} < u_l^{TC_\delta}$ follows similarly using (11) and (13).[16] ■

6 Appendix B: Extensions

6.1 Continuous Effort and $N > 2$ Outcome Levels

Here I consider the continuous version of the model (as in Rogerson (1985)) and show that the main result is still valid. Suppose, the agent chooses an effort level, e, from an interval, say $[0,1]$. The outcome can be one of the $N > 2$ values, $\{q_1, q_2, ..., q_N\}$. Let $p_j(e)$ denote the probability of q_j being realized given that the agent has exerted effort level e. Let the cost of effort e be given by $\psi(e)$ with $\psi' > 0$, $\psi(0) = 0$ and $\psi(e) > 0$ for all $e \in (0,1]$. Let, again, $u(\cdot)$ be the wage utility function with $u' > 0, u'' < 0$ and with an inverse function $h(\cdot)$. Then, given that the principal wants to implement $e = e^* \in (0,1)$ in each period[17], the principal's problem is given by

$$\min_{\{w_i, w_{ij}\}_{i,j\in\{1,2,...,N\}}} \sum_{i=1}^{N} p_i(e^*)[w_i + \delta_P[\sum_{j=1}^{N} p_j(e^*)w_{ij}]$$

subject to

$$\begin{array}{ll} (IR) & \sum_{i=1}^{N} p_i(e^*)[u(w_i) - \psi(e^*) + \delta[\sum_{j=1}^{N} p_j(e^*)u(w_{ij}) - \psi(e^*)]] \geq 0 \\ (IC_1) & e = e^* \in argmax_e \sum_{i=1}^{N} p_i(e)[u(w_i) - \psi(e) + \delta\beta[\sum_{j=1}^{N} p_j(e)u(w_{ij}) - \psi(e)]] \\ (IC_2) & e = e^* \in argmax_e \sum_{j=1}^{N} p_j(e)u(w_{ij}) - \psi(e)]] \text{ for } i \in \{1,2,...,N\} \end{array}$$

The necessary condition for IC_1 is given by

$$\sum_{i=1}^{N} p_i'(e^*)[u(w_i) - \psi'(e^*) + \delta\beta[\sum_{j=1}^{N} p_j'(e^*)u(w_{ij}) - \psi'(e^*)]] = 0$$

[16]Suppose $u_l^{SO} \geq u_l^{TC_\delta}$. Then (11) implies $Eu_l^{SO} < Eu_l^{TC_\delta}$,whereas (13) implies $(g \circ C_2')(Eu_l^{SO}) > (g \circ C_2')(Eu_l^{TC_\delta})$ which in turn implies $Eu_l^{SO} > Eu_l^{TC_\delta}$. Therefore $u_l^{SO} < u_l^{TC_\delta}$.

[17]Here, I consider an interior effort level in order to have the first order conditions hold with equality, which makes the analysis easier.

or

$$\sum_{i=1}^{N} p_i'(e^*)[u(w_i) + \delta\beta[\sum_{j=1}^{N} p_j'(e^*)u(w_{ij})]] = \psi'(e^*)(1+\delta\beta)$$

And, the necessary condition for IC_2 for a given $i \in \{1, 2, ..., N\}$ is given by

$$\sum_{j=1}^{N} p_j'(e^*)u(w_{ij}) = \psi'(e^*)$$

Replacing $u(w_i)$ with u_i, $u(w_{ij})$ with u_{ij}, w_i with $h(u_i)$, and w_{ij} with $h(u_{ij})$, the principal's problem becomes

$$\min_{\{u_i, u_{ij}\}_{i,j\in\{1,2,...,N\}}} \sum_{i=1}^{N} p_i(e^*)[h(u_i) + \delta_P[\sum_{j=1}^{N} p_j(e^*)h(u_{ij})]$$

subject to

$$\begin{array}{ll} (IR) & \sum_{i=1}^{N} p_i(e^*)[u_i + \delta[\sum_{j=1}^{N} p_j(e^*)u_{ij}]] \geq \psi(e^*)(1+\delta) \\ (IC_1) & \sum_{i=1}^{N} p_i'(e^*)[u_i + \delta\beta[\sum_{j=1}^{N} p_j'(e^*)u_{ij}]] = \psi'(e^*)(1+\delta\beta) \\ (IC_2) & \sum_{j=1}^{N} p_j'(e^*)u_{ij} = \psi'(e^*) \text{ for } i \in \{1, 2, ..., N\} \end{array}$$

For a given first period output level $q_i \in \{q_1, ..., q_N\}$, the agent's continuation payoff will be $\sum_j p_j(e^*)u_{ij} - \psi(e^*)$. If the agent has been promised Eu_i for the second period when the first period output realization is q_i, then $\{u_{ij}\}_j$ are defined to be

$$\sum_{j=1}^{N} p_j(e^*)u_{ij} - \psi(e^*) = Eu_i \text{ for } i \in \{1, 2, ..., N\}$$

Once the principal promises the agent a utility of Eu_i, the continuation of the optimal contract for the second period will be given by the solution to the following problem

$$\min_{\{u_{ij}\}_{j\in\{1,2,...,N\}}} \sum_{j=1}^{N} p_j(e^*)h(u_{ij})$$

subject to

$$\sum_{j=1}^{N} p_j(e^*)u_{ij} - \psi(e^*) = Eu_i$$

$$\sum_{j=1}^{N} p_j'(e^*)u_{ij} = \psi'(e^*) \text{ for } i \in \{1, 2, ..., N\}$$

Denote the cost of implementing the effort, $e = e^*$, in the second period, given that the promised second period utility is Eu_i, by $C_2(Eu_i)$. Then,

$$C_2(Eu_i) = \sum_{j}^{N} p_j(e^*)h(u_{ij})$$

Now the principal's problem can be reduced to

$$\min_{\{u_i, Eu_i\}_{i\in\{1,2,...,N\}}} \sum_{i}^{N} p_i(e^*)[h(u_i) + \delta_P C_2(Eu_i)]$$

subject to

$$\begin{array}{ll} (IR) & \sum_i p_i(e^*)[u_i + \delta Eu_i] \geq \psi(e^*) \\ (IC) & \sum_{i=1}^{N} p_i'(e^*)[u_i + \delta\beta[Eu_i]] = \psi'(e^*) \end{array}$$

Attaching λ to IR and μ to IC we have the following first order conditions with respect to u_i and Eu_i, respectively.

$$h'(u_i) = \mu \frac{p_i'(e^*)}{p_i(e^*)} + \lambda \tag{14}$$

$$\frac{\delta_P}{\delta} C_2'(Eu_i) = \beta\mu \frac{p_i'(e^*)}{p_i(e^*)} + \lambda \tag{15}$$

(14) and (15) imply that

$$h'(u_i) = \frac{\delta_P}{\delta} C_2'(Eu_i) + (1-\beta)\mu \frac{p_i'(e^*)}{p_i(e^*)} \tag{16}$$

The equivalent of Proposition 1 is still valid under a nontriviality condition which ensures that $p_k(e)$ is not constant at $e = e^*$ for some $k \in N$. Since $\sum_j p_j(e) = 1$ for each $e \in [0, 1]$, this condition implies that there exists an outcome q_i such that $p_i'(e = e^*) > 0$ and another outcome q_k such that $p_k'(e = e^*) < 0$. Under this condition, it is easy to see that for those outcome levels with $p_i'(e = e^*) > 0$, we have

$$h'(u_i^{SO}) > \frac{\delta_P}{\delta} E_j(h'(u_{ij}^{SO}))$$

and for those outcome levels with $p'_k(e = e^*) < 0$, we have

$$h'(u_k^{SO}) \quad < \quad \frac{\delta_P}{\delta} E_j(h'(u_{kj}^{SO}))$$

Thus, Proposition 1 still holds.

Now, let's look at the evolution of wages over time. Here, I assume $\delta_P = \delta$. Using the definition $h(u(w_i)) = w_i$ where $h = u^{-1}$. Then, $h'(u_i) = \frac{1}{u'(w_i)}$. Then the equations above become

$$\frac{1}{u'(w_i^{SO})} \quad > \quad \sum_{j=1}^{N} \frac{p_j(e^*)}{u'(w_{ij}^{SO})}$$

for those outcome levels with $p'_i(e = e^*) > 0$. And for those outcome levels with $p'_k(e = e^*) < 0$, we have

$$\frac{1}{u'(w_k^{SO})} \quad < \quad \sum_{j=1}^{N} \frac{p_j(e^*)}{u'(w_{kj}^{SO})}$$

Suppose $\frac{1}{u'}$ is convex. Then, by Jensen Inequality, for any i, we have

$$\sum_{j=1}^{N} \frac{p_j(e^*)}{u'(w_{ij}^{SO})} \geq \frac{1}{u'(\sum_{j=1}^{N} p_j(e^*) w_{ij}^{SO})}$$

Thus, for i such that $p'_i(e = e^*) > 0$, we get

$$\frac{1}{u'(w_i^{SO})} \quad > \quad \frac{1}{u'(\sum_{j=1}^{N} p_j(e^*) w_{ij}^{SO})}$$

which implies $w_i > \sum_{j=1}^{N} p_j(e^*) w_{ij}^{SO}$ by the fact that $\frac{1}{u'}$ is increasing. This is the same result Rogerson(1985) gets when $\frac{1}{u'}$ is convex, but for only those outcomes with $p'_i(e = e^*) > 0$. Thus, conditional on period 1 outcome being among the better outcomes, the period 1 wage is higher than the expected period 2 wage. However, this is not necessarily the case if the period 1 outcome is such that $p'_k(e = e^*) < 0$. A similar argument also works for the case where $\frac{1}{u'}$ is concave. In that case, the period 1 wage will be smaller than the expected period 2 wage whenever the period 1 outcome is among the bad ones.[18]

[18] Here I refer to the outcomes with $p'_i(e = e^*) > 0$ the good ones and the outcomes with $p'_k(e = e^*) < 0$ the bad ones.

6.2 Effort choice with 3 periods

In the main text, although the model was a 3 period model, the effort choice was made in only the 2nd and the 3rd periods. This causes β to disappear from the individual rationality constraint. If, however, there is also an effort choice in the 1st period, then β would not disappear from the individual rationality constraint. Here I show that this is not an issue, that is, the results with a sophisticated agent are robust to this modification.

The principal's cost minimization problem, given that she wants to implement $e = 1$ in each period, is given by, after using $h(\cdot)$, the inverse function of $u(\cdot)$ and carrying out the last period's incentive problem,

$$\min_{\{u_i, u_{ij}, EU_{ij}\}_{i,j\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1^2[h(u_{hh}) + \delta_P C_3(EU_{hh})] + p_1(1-p_1)[h(u_{hl}) + \delta_P C_3(EU_{hl})]$$
$$(1-p_1)p_1[h(u_{lh}) + \delta_P C_3(EU_{lh})] + (1-p_1)^2[h(u_{ll}) + \delta_P C_3(EU_{ll})]]$$

subject to

$$(IR) \quad p_1 u_h + (1-p_1)u_l + \delta\beta[p_1^2[u_{hh} + \delta EU_{hh}] + p_1(1-p_1)[u_{hl} + \delta EU_{hl}]$$
$$+(1-p_1)p_1[u_{lh} + \delta EU_{lh}] + (1-p_1)^2[u_{ll} + \delta EU_{ll}]] \geq (1+\delta\beta)\psi$$

$$(IC_1) \quad u_h - u_l + \delta\beta[p_1[(u_{hh} - u_{lh}) + \delta(EU_{hh} - EU_{lh})] + (1-p_1)[(u_{hl} - u_{ll}) + \delta(EU_{hl} - EU_{ll})]] \geq \frac{\psi}{p_1 - p_0}$$

$$(IC_{2,i}) \quad u_{ih} - u_{il} + \delta\beta(EU_{ih} - EU_{il}) \geq \frac{\psi}{p_1 - p_0} \quad \forall i \in \{h, l\}$$

where EU_{ij} is the promised expected utility for the third period to the agent when the first period and second period realizations are i and j, respectively. Then, forming the Lagrangian with λ attached to the IR, μ to the IC_1, μ_i to the $IC_{2,i}$, and looking at the first order conditions, for instance, for u_{hh} and EU_{hh}, we get

$$\delta_P p_1^2 h'(u_{hh}) - \lambda\delta\beta p_1^2 - \mu\delta\beta p_1 - \mu_h = 0$$

$$\delta_P^2 p_1^2 C_3'(EU_{hh}) - \lambda\delta^2\beta p_1^2 - \mu\delta^2\beta p_1 - \mu_h\delta\beta = 0$$

where $C_3(EU_{ij})$ is the cost of implementing the high effort level in the third period, given that the promised third period utility is EU_{ij}. These two conditions together imply

$$h'(u_{hh}) = C_3'(EU_{hh}) + \frac{\mu_h(1-\beta)}{\delta p_1^2}$$

which is similar to the equation (7), assuming $\delta_P = \delta$. Also, very similarly, from the first order conditions with respect to u_{ll} and EU_{ll}, we get

$$h'(u_{ll}) = C_3'(EU_{ll}) - \frac{\mu_l(1-\beta)}{\delta(1-p_1)^2}$$

which is similar to the equation (8), assuming $\delta_P = \delta$. Now it's easy to see that the main result still holds across second and third periods.

6.3 Access to the Credit Market

In this section I provide an analysis, close to the one in Rogerson (1985), where I allow the agent to be able to save and borrow. There are three periods, $t = 0, 1, 2$, first one ($t = 0$) being the contracting stage and in the next two the agent makes an effort choice, as in section 2.1 in the main text. Also, I assume the effort is picked from a set $[0, 1]$, as in section 6.1 above. Suppose that after the outcome is realized at the end of $t = 1$, the agent is allowed to save or borrow at the interest rate r. Let w be the Pareto optimal wage contract that implements effort level $e = e^* \in (0, 1)$ in each period, if the agent is not allowed to save or borrow. When the agent is allowed to save and borrow, let the savings/borrowings at the end of $t = 1$ be denoted by s_1. (Since there is no further period, the savings/borrowings at the end of $t = 1$ is 0). Without loss of generality, a positive s_1 represents savings and a negative s_1 represents borrowings. Then, the discounted expected utility for the agent is given by

$$u(w_i - s_1) + \delta\beta \sum_{j=1}^{N} p_j(e^*) u(w_{ij} + (1+r)s_1)$$

The marginal discounted expected utility with respect to saving s_1 is then given by

$$-u'(w_i - s_1) + \delta\beta(1+r) \sum_{j=1}^{N} p_j(e^*) u'(w_{ij} + (1+r)s_1)$$

This expression is not necessarily positive at $s_1 = 0$. To see this, note that

$$\delta\beta(1+r) \sum_{j=1}^{N} p_j(e^*) u'(w_{ij}) \geq \frac{\delta\beta(1+r)}{\sum_{j=1}^{N} \frac{p_j(e^*)}{u'(w_{ij})}} \tag{17}$$

which follows from the fact that

$$\sum_{j=1}^{N} p_j \alpha_j \geq \frac{1}{\sum_{j=1}^{N} \frac{p_j}{\alpha_j}}$$

where α_j's are nonnegative numbers and p_j's are probabilities adding up to one.

Now, from section 6.1 we know that for those outcome levels with $p'_j(e = e^*) > 0$, we have

$$h'(u_i^{SO}) \quad > \quad \frac{\delta_P}{\delta} E_j(h'(u_{ij}^{SO}))$$

which is equivalent to

$$\frac{\delta(1+r)}{u'(w_i)} > \sum_{j=1}^{N} \frac{p_j}{u'(w_{ij})}$$

Then, (17) implies that

$$\delta\beta(1+r)\sum_{j=1}^{N} p_j(e^*)u'(w_{ij}) \geq \frac{\delta\beta(1+r)}{\sum_{j=1}^{N} \frac{p_j(e^*)}{u'(w_{ij})}} > \frac{\delta\beta(1+r)}{\frac{\delta(1+r)}{u'(w_i)}} = \beta u'(w_i)$$

Thus, $\delta\beta(1+r)\sum_{j=1}^{N} p_j(e^*)u'(w_{ij} + (1+r)s_1) - u'(w_i - s_1)$ is not necessarily positive. For $\beta = 1$, this expression is going to be positive and the agent will be saving. This is the case in Rogerson (1985), thus it becomes a special case of the model here. But for low values of β, this expression might be negative, hence the agent might chose to borrow. This is intuitive because, when β is low the agent is more time-inconsistent and would be willing to consume more today.

6.4 Comparison with an Alternative Time-consistent Agent

In the analysis in the text I compared the sophisticated agent with two time-consistent agents, one with discount factor δ, and the other with the discount factor $\delta\beta$. Here, I will make one more comparison. Consider a time-consistent agent with a discount factor Δ where $\Delta + \Delta^2 = \delta\beta + \delta^2\beta$.[19] Thus, the time-consistent agent I consider here has the same average discount factor with the sophisticated agent.

[19] See Chade et.al (2008)

Proposition 5 *The expected profit from SO is lower than that from TC_Δ, whenever (i) the expected second period promised utility is positive for TC_Δ and $\delta \geq \Delta(2-\beta)$,*[20] *or (ii) the expected second period promised utility is negative for SO.*

Proof. The individual rationality and incentive constraints are as follows

$$\begin{array}{ll} (IR) & p_1 u_h + (1-p_1)u_l + \alpha[p_1 Eu_h + (1-p_1)Eu_l] \geq \psi \\ (IC) & u_h - u_l + \gamma\alpha(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0} \end{array}$$

where the case for a sophisticated agent with $(1, \delta\beta, \delta^2\beta)$ is represented by $(\alpha, \gamma) = (\delta, \beta)$ and the case for a time-consistent agent with the discount factor Δ is represented by $(\alpha, \gamma) = (\Delta, 1)$. Moving from $(\alpha, \gamma) = (\Delta, 1)$ to $(\alpha, \gamma) = (\delta, \beta)$, the change in the expected cost of implementing high effort in both periods is approximately

$$(\delta - \Delta)\frac{\partial EC}{\partial \alpha}|_{(\alpha,\gamma)=(\Delta,1)} + (\beta - 1)\frac{\partial EC}{\partial \gamma}|_{(\alpha,\gamma)=(\Delta,1)}$$

Attaching Lagrange multiplier λ to the individual rationality constraint, and μ to the incentive compatibility, we get

$$(\delta - \Delta)[\lambda EU_2 + \mu(Eu_h - Eu_l)] - (1-\beta)[\mu\Delta(Eu_h - Eu_l)]$$

where $EU_2 = p_1 Eu_h + (1-p_1)Eu_l$. The change then becomes

$$(\delta - \Delta)\lambda EU_2 + [(\delta - \Delta) - \Delta(1-\beta)]\mu(Eu_h - Eu_l)$$

that is,

$$(\delta - \Delta)\lambda EU_2 + [\delta - \Delta(2-\beta)]\mu(Eu_h - Eu_l)$$

Note that $\delta > \Delta$ and $Eu_h > Eu_l$. Thus, whenever $EU_2 > 0$,[21] and $\delta \geq \Delta(2-\beta)$, the above expression is positive, thus, the cost of implementing high effort in both periods is larger with the sophisticated agent than with the time-consistent agent with discount factor Δ.

Part (ii) follows from Proposition 4: Note that when the principal is facing a time-consistent agent, she is better off when the agent has a higher discount factor. Thus, the

[20] This condition is satisfied for a large set of parameters: For instance, whenever $\beta < 0.64$, this is satisfied for any δ. Note that Δ is a function of δ and β.

[21] This is calculated at $(\alpha, \gamma) = (\Delta, 1)$, that is, $EU_2 > 0$ is assumed for the time-consistent agent with a discount factor Δ.

expected profit from TC_Δ is higher than the expected profit from $TC_{\delta\beta}$, since $\Delta > \delta\beta$. But when EU_2 is negative for SO, the expected profit from $TC_{\delta\beta}$ is higher than that from SO by Proposition 4. Thus, the result follows. ■

6.5 Relaxing the Full Commitment Assumption

In this section, I relax the assumption of full commitment by the agent. That is, at the end of $t = 1$,[22] the agent may quit the contract if he realizes that the continuation expected profit is negative. Then, the optimal contract that would make sure that the agent would not quit at the end of $t = 1$, solves the following problem of the principal:

$$\min_{\{u_i, Eu_i\}_{i\in\{h,l\}}} p_1 h(u_h) + (1-p_1)h(u_l) + \delta_P[p_1 C_2(Eu_h) + (1-p_1)C_2(Eu_l)]$$

subject to

$$\begin{array}{ll} (IR) & p_1 u_h + (1-p_1)u_l + \delta[p_1 Eu_h + (1-p_1)Eu_l] \geq \psi \\ (IC) & u_h - u_l + \beta\delta(Eu_h - Eu_l) \geq \frac{\psi}{p_1 - p_0} \\ (NC) & p_1 Eu_h + (1-p_1)Eu_l \geq 0 \end{array}$$

Here the NC (no commitment) condition makes sure that the agent does not quit at the end of $t = 1$. Attaching λ to IR, μ to IC and θ to NC, we have the following first order conditions with respect to u_h, u_l, Eu_h and Eu_l, respectively.

$$h'(u_h) = \frac{\mu}{p_1} + \lambda \tag{18}$$

$$h'(u_l) = -\frac{\mu}{1-p_1} + \lambda \tag{19}$$

$$\frac{\delta_P}{\delta} C_2'(Eu_h) = \frac{\beta\mu}{p_1} + \lambda + \frac{\theta}{\delta} \tag{20}$$

$$\frac{\delta_P}{\delta} C_2'(Eu_l) = -\frac{\beta\mu}{1-p_1} + \lambda + \frac{\theta}{\delta} \tag{21}$$

(18) and (19) imply that

$$\lambda = p_1 h'(u_h) + (1-p_1)h'(u_l) \tag{22}$$

$$\mu = p_1(1-p_1)(h'(u_h) - h'(u_l)) \tag{23}$$

[22] Here, I stick to the original discrete 3-period model with $t = 0, 1, 2$ where $t = 0$ is the contracting stage with no effort choice is made.

(18) and (20) imply that

$$h'(u_h) = \frac{\delta_P}{\delta} C_2'(Eu_h) - \frac{\theta}{\delta} + \frac{\mu}{p_1}(1-\beta) \tag{24}$$

(19) and (21) imply that

$$h'(u_l) = \frac{\delta_P}{\delta} C_2'(Eu_l) - \frac{\theta}{\delta} - \frac{\mu}{1-p_1}(1-\beta) \tag{25}$$

(20) and (21) imply that

$$\frac{\delta_P}{\delta}(p_1 C_2'(Eu_h) + (1-p_1)C_2'(Eu_l)) = \lambda + \frac{\theta}{\delta} \tag{26}$$

Note that $\theta > 0$, because if $\theta = 0$, then NC would not bind and the solution with the full commitment assumption would also solve this problem. We also know that $\lambda > 0$ and $\mu > 0$. Then, (22) and (26) together imply that

$$p_1 h'(u_h) + (1-p_1)h'(u_l) < \frac{\delta_P}{\delta}(p_1 C_2'(Eu_h) + (1-p_1)C_2'(Eu_l)) \tag{27}$$

Also, (25) implies that

$$h'(u_l) < \frac{\delta_P}{\delta} C_2'(Eu_l) \tag{28}$$

Thus, neither $h'(u_h) \geq \frac{\delta_P}{\delta} C_2'(Eu_h)$ nor $h'(u_h) < \frac{\delta_P}{\delta} C_2'(Eu_h)$ is ruled out. If it's the former case than the Propositions 1 and 2 in the main section under full commitment assumption are still valid. This is the case where it's easier to keep the agent in the contract, hence no commitment assumption does not change the tradeoff between payments today and tomorrow much. But if it's the latter, then it means that in the optimal contract, given any first period output realization, the marginal cost of rewarding the sophisticated agent for this output realization in the first period is lower than the expected marginal cost of promising these awards in the second period. This is where the no commitment assumption kicks in: The wages should be relatively higher in the second period to ensure that the agent does not quit the contract. Thus, it's more costly to reward the agent in the second period to provide the necessary incentives for both high effort and for not quitting.

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