TÜBİTAK

Project Title: Regularity Properties of Dispersive Partial Differential Equations

Project Summary

Given a dispersive partial differential equation (PDE) on a certain domain, it is one of the major research directions to analyze the behavior of solutions with regards to the initial and boundary data. More precisely, main aim is to prove local and global in time well-posedness and possibly that the nonlinear solution belongs to a smoother (better) function space than that of the initial data. This smoothness is formulated in a particular way and investigates the smoothness of the difference between the solution and the solution of the linear part of the equation, and called nonlinear smoothing. The smoothing effect, once proven, yields an elegant way to prove, improve or simplify the well-posedness results. By well-posedness is meant the existence, uniqueness and the continuous dependence on initial data of the solution.

In case of $\mathbb{R}^d$ and $\mathbb{T}^d$ (d-dimensional torus), there are several papers in which the smoothing effect was proved and quantified, namely, the upper bound of the derivative gain was formulated in terms of the smoothness of the initial data. We give some sample results in the literature survey. Investigation of initial-boundary value problems is a more open research field. More precisely, if the space domain is semi-bounded (such as half-line or upper half-plane) or bounded (such as an interval), we have to impose boundary condition(s) which also contributes to the evolution at various levels. This creates a lot of complications which have to be handled carefully. Moreover, the choice of the function space that the boundary data belongs to plays a crucial role in establishing the required estimates.

This proposed study is about utilizing harmonic analysis techniques to resolve some of the smoothing and consequently well-posedness issues for certain dispersive PDEs. Such techniques have been successfully applied on $\mathbb{R}^d$ and $\mathbb{T}^d$ by many mathematicians since early 1990s following the seminal papers of Bourgain. From mid 2000s and onwards, similar methods have been used to prove well-posedness of dispersive PDEs on half or fully bounded domains. Recently, also nonlinear smoothing estimates have been obtained for the Schrödinger equation with cubic nonlinearity, Zakharov system and derivative nonlinear Schrödinger equation on the half-line. These estimates simplified or improved the previous well-posedness theories of the initial-boundary value problems of the relevant equations. In the light of our previous work, we plan to examine initial-boundary value problems of the Kadomtsev-Petviashvili and Kuznetsov-Zakharov equations as examples. We also contemplate to handle some important PDEs such as the Davey-Stewartson and its generalized version first on $\mathbb{R}^d$ and $\mathbb{T}^d$. It must be noted that, despite the similarities, all of these equations require different approaches and the methods for establishing the estimates could vary dramatically in difficulty and complexity level.

Keywords: dispersive equation, well-posedness, nonlinear smoothing, initial-boundary value problem, Fourier restriction method