

Algebra II

HOMEWORK 7 - DUE MAY 11, 2026

1. Let I be an ideal of a ring R . Show that I is primary if and only if the only zero divisors of R/I are nilpotent elements.
2. Find a ring R and an ideal $I \subseteq R$ such that $\text{Rad}(I)$ is prime, but I not primary.
3. Let R be a ring such that for any $x \in R$ there is $n > 0$ such that $x^n = x$. Show that every prime ideal of R is a maximal ideal.
4. Let R be an integrally closed integral domain with field of fractions K . Let t be transcendental over K . Show that $R[t]$ is also integrally closed.