

Algebra II

HOMEWORK 6 - DUE APRIL 29, 2026, 11:00

1. Let $K|k$ be an algebraic extension and $L|k$ be an arbitrary extension. Show that $K \perp_k L$ if and only if for every subfield E of K that is finitely generated over k we have $[E : k] = [EL : L]$.
2. Construct a tower $k \subseteq K \subseteq L$ such that $L|k$ is regular, but $L|K$ is not.
3. (a) Let K and L be arbitrary extensions of k . Show that there are an extension F of k and field embeddings $\sigma : K \rightarrow F$ and $\tau : L \rightarrow F$ such that $\sigma(K) \perp_k \tau(L)$.
(b) Show that the result above is not correct when \perp is replaced by \perp .