

Algebra II

HOMEWORK 1 - DUE FEBRUARY 18, 2026

1. Let $f(x) = x^3 - 2x - 2 \in \mathbb{Q}[x]$ and let $\alpha \in \mathbb{C}$ be a root of f . Write $\frac{1+\alpha}{1+\alpha+\alpha^2}$ as a \mathbb{Q} -linear combination of $1, \alpha, \alpha^2$.
2. Let T be an indeterminate and let $K = \mathbb{Q}(T)$ be the field of rational functions in T with rational coefficients. Let α be a root of $f(x) = x^2 - T \in K[x]$. Write the multiplicative inverse of $1 + \alpha$ as a K -linear combination of 1 and α .
3. Let $E|F$ be an algebraic extension and let R be a subring of E containing F . Show that R is indeed a field.
4. Let T be an indeterminate (over \mathbb{R}). Calculate $[\mathbb{R}(T) : \mathbb{R}(T + \frac{1}{T})]$.